

FLUID MECHANICS

I_{ST}

* Review of fluid properties:-

⇒ The study of fluids at rest is called fluid statics.

⇒ The study of fluids in motion, where pressure forces are not considered, is called fluid kinematics & if the pressure forces are also considered for the fluids in motion, that branch of science is called fluid dynamics.

* Properties of fluids:-

1. Density or mass density:- Density or mass density of a fluid is defined as the ratio of the mass of a fluid to its volume. Unit $\rightarrow \text{kg/m}^3$. The density of liquids may be considered as constant while that of gases changes with the variation of press. & temperature.

$$\rho = \frac{\text{Mass of fluid}}{\text{Volume of fluid}}$$

2. Specific Weight or Weight Density:- It is the ratio b/w the weight of a fluid to its volume. Thus weight per unit volume of a fluid is called weight density & it is denoted by the symbol w

$$w = \frac{\text{Weight of fluid}}{\text{Vol}^m \text{ of fluid}}$$

$$w = \frac{\text{mass} \times \text{acceleration due to gravity}}{\text{Vol}^m \text{ of fluid}}$$

$$w = \rho g$$

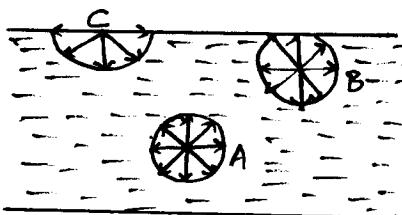
3. Specific Volume:- Specific volume of a fluid is defined as the volume of a fluid occupied by a unit mass or volume per unit mass of a fluid is called sp. vol^m

$$\text{Sp. Vol}^m = \frac{1}{\rho}$$

h. Specific Gravity :- It is defined as the ratio of the weight density of a fluid to the weight density of standard fluid. For liquid std. fluid is water & for gases std. fluid is air.

* Surface Tension & Capillary:-

Surface tension is defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface b/w two immiscible liquids such that the contact surface behaves like a membrane under tension. The magnitude of this force per unit length of the free surface will have the same value as the surface energy per unit area. It is denoted by Greek letter σ . In MKS unit, it is expressed as kgf/m in SI unit as N/m.



* Surface Tension on liquid Droplet :- Consider a small spherical droplet of a liquid of radius 'r'. On the entire surface of the droplet, the tensile force due to surface tension will be acting.

Let, σ = Surface tension of the liquid

P = Pressure intensity inside the droplet

d = Dia of droplet.

Let the droplet is cut into two halves. The forces acting on one half will be.

(i) Tensile force due to surface tension acting around the circumference of the cut portion as shown in fig. & this is equal to.

$$= \sigma \times \text{Circumference}$$

$$= \sigma \times \pi d$$



(ii) Pressure force on the area $\frac{\pi}{4}d^2 = P \times \frac{\pi}{4}d^2$

The two forces will be equal & opposite under equilibrium condition i.e.

$$\Rightarrow P \times \frac{\pi}{4} d^2 = \sigma \times \pi d$$

$$\Rightarrow P = \frac{4\sigma}{d}$$

2. Surface Tension on a Hollow Bubble :- A hollow bubble like a soap bubble in air has two surfaces in contact with air, one inside & other outside. Thus two surfaces are subjected to surface tension. In such a case we have

$$P \times \frac{\pi}{4} d^2 = 2(\sigma \times \pi d)$$

$$\boxed{P = \frac{8\sigma}{d}}$$

* Surface Tension on a Liquid Jet :- Consider a liquid jet of dia 'd' & length 'L' as shown in fig

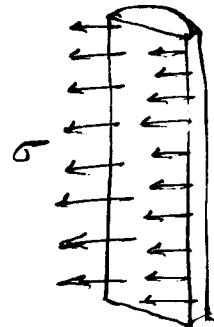
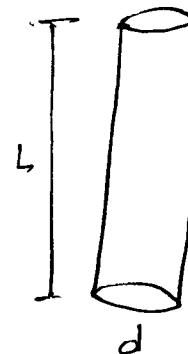
$$\Rightarrow \text{force due to pressure} = P \times \text{area of semi jet}$$

$$= P \times L \times d$$

$$\text{force due to surface tension} = \sigma \times 2L$$

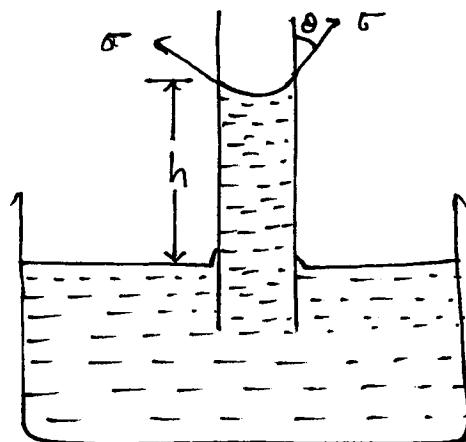
$$\therefore P \times L \times d = \sigma \times 2L$$

$$\boxed{P = \frac{2\sigma}{d}}$$



* Capillarity :- It is defined as a phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid. The rise of liquid surface is known as capillary rise & the fall of the liquid surface is known as capillary depression. Its value depends upon the specific weight of the liquid, diameter of the tube & surface tension of the liquid.

* Expression for Capillary Rise :-



→ Capillary Rise

d = diameter of tube

h → height of the liquid in the tube, under a state of equi-

⇒ The force at the surface of the liquid in the tube is due to surface tension (σ)

θ = angle of contact b/w liquid & glass tube.

⇒ The weight of liquid of height h in the tube =

$$= (\text{Area of tube} \times h) \times \rho \times g$$

$$= \frac{\pi}{4} d^2 h \rho g$$

⇒ Vertical component of the surface tensile force

$$= (\sigma \times \text{circumference}) \times \cos \theta$$

$$= \sigma \times \pi d \times \cos \theta$$

∴ For equilibrium

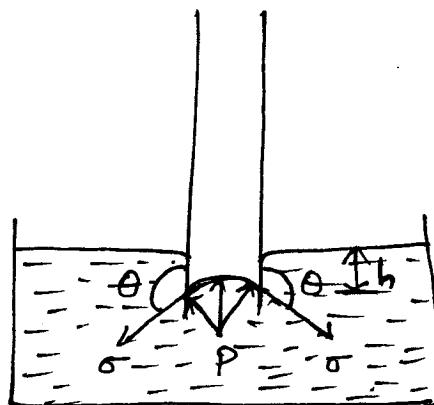
$$\frac{\pi}{4} d^2 h \rho g = \sigma \pi d \cos \theta$$

$$\therefore h = \frac{4\sigma \cos \theta}{\rho g d}$$

$\theta \rightarrow$ approximately zero

$$\therefore h = \frac{4\sigma}{\rho g d}$$

* Expression for Capillary fall :-



h = height of depression in tube

1. \rightarrow Due to surface tension acting in the downward direction & is equal to $= \sigma \times \pi d \times \cos \theta$ ————— 1

2. \rightarrow Due to hydrostatic force acting upward & is equal to intensity of pressure at a depth $h \times \text{Area}$

$$= P \times \frac{\pi}{4} d^2 \quad \therefore P = \rho g h$$

$$\therefore = \rho g h \times \frac{\pi}{4} d^2$$

At equilibrium

$$\sigma \pi d \cos \theta = \rho g h \frac{\pi}{4} d^2$$

$$\therefore h = \frac{4 \sigma \cos \theta}{\rho g d}$$

* Compressibility & Bulk Modulus:-

Compressibility is the reciprocal of the bulk modulus of elasticity K which is defined as the ratio of compressive stress to volumetric strain.

Consider a cylinder fitted with a piston as shown in fig.

Let V = Volume of a gas enclosed in the cylinder.

P = Pressure of the gas when vol^m is V

Let the pressure is increased to $P+dP$, the vol^m of gas decreases from V to $V-dV$.

The increase in pressure = dP Kgf/m²

Decrease in volume = dV

∴ Volumetric strain = $\frac{-dV}{V}$

→ ve sign means the volume decreases with increase of pressure

$$\therefore \text{Bulk modulus } K = \frac{\text{Increase of pressure}}{\text{Volumetric strain}}$$

$$= \frac{-\frac{dP}{dV}}{V} = -\frac{dP}{dV} V$$

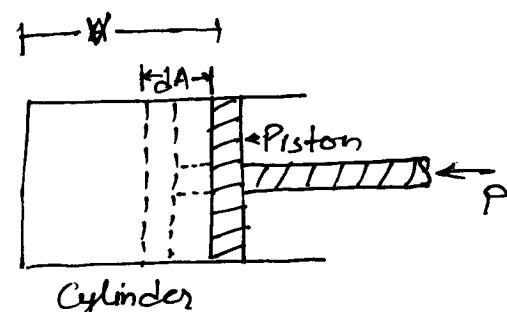
$$\therefore K = \boxed{\frac{-dP}{dV} V}$$

* Fluid Pressure at a point:-

Consider a small area dA in large mass of fluid. If the fluid is stationary, then the force exerted by the surrounding fluid on the area dA will always be perpendicular to the surface dA .

Let dF is the force acting on the area dA in the normal direction. Then the ratio of $\frac{dF}{dA}$ is known as the intensity of pressure or simply pressure & this ratio is represented by P . Hence mathematically the pressure at a point in a fluid at rest is

$$\Rightarrow \boxed{P = \frac{dF}{dA}}$$



If the force F is uniformly distributed over the area (A), then pressure at any point is given by

$$\Rightarrow P = \frac{F}{A} \quad \frac{\text{N}}{\text{m}^2}$$

* PASCAL'S LAW :-

It states that the pressure or intensity of pressure at a point in a static fluid is equal in all directions. This is proved as:-

The fluid element is of very small dimensions i.e $\Delta x, \Delta y, \Delta s$

Consider an arbitrary fluid element of wedge shape in a fluid mass at rest as shown in fig. Let the width of the element perpendicular to the plane of paper is unity & P_x . P_y & P_z are the pressures acting on the face AB , AC & BC respectively. Let $\angle ABC = \theta$. Then the force acting on the element are!

1. Pressure force normal to the surface.
2. Weight of element in vertical direction.

The force on the faces are

$$\begin{aligned}\text{Force on the face } AB &= P_x \times \text{Area of face } AB \\ &= P_x \times \Delta y \times 1\end{aligned}$$

$$\text{Similarly on the } AC = P_y \Delta x \cdot 1$$

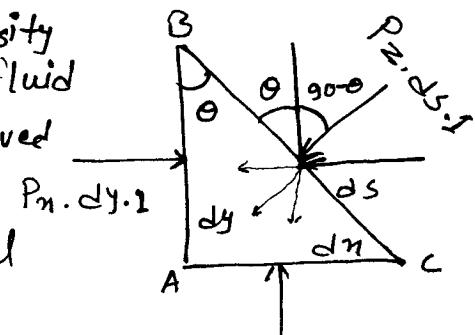
$$\text{on } BC = P_z \Delta s \cdot 1$$

$$\begin{aligned}\therefore \text{Weight of the element} &= \text{Mass of element} \times g \\ &= \text{Volume} \times \rho \times g \\ &= \frac{1}{2} AB \times AC \times 1 \times \rho \times g\end{aligned}$$

Resolving the force in x direction

$$P_x \times \Delta y - P_z \Delta s \sin(90 - \theta) = 0$$

$$\therefore P_x \Delta y - P_z \Delta s \cos \theta = 0$$



* \Rightarrow But from fig

$$ds \cos \theta = AB = dy$$

$$\therefore P_n dy - P_z dy = 0$$

$$\therefore P_n = P_z \quad \text{--- (1)}$$

Similarly, resolving the force in y direction, we get

$$P_y dn - P_z ds \cos(g_0 - \theta) - \frac{dn dy}{2} sg = 0$$

$$\therefore P_y dn - P_z ds \sin \theta - \frac{dn dy}{2} sg = 0$$

But $ds \sin \theta = dn$ and also the element is very small & hence the weight is negligible.

$$\therefore P_y dn - P_z dn = 0$$

$$P_y = P_z \quad \text{--- (2)}$$

$$\therefore \boxed{P_n = P_y = P_z},$$

Since the choice of fluid element was completely arbitrary, which means pressure at any point is the same in all directions.

* Pressure variation in a fluid at rest / static :-

The pressure at any point in a fluid at rest is obtained by the Hydrostatic Law which states that the rate of increase of pressure in a vertically downward direction must be equal to the sp. weight of the fluid at that point.

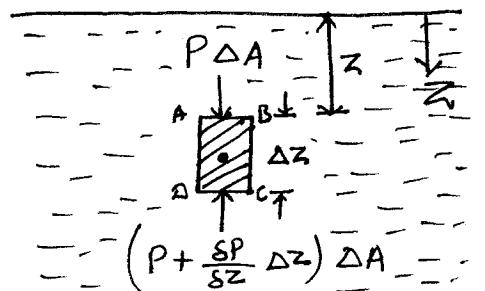
Consider a small fluid element as shown in fig

Let ΔA = Gross sectional area

Δz = Height of fluid element

P = Pressure on face AB

Z = Distance of fluid element from free surface.



* The force acting on the fluid element are.

1. Pressure force on AB = $P \times \Delta A \downarrow$

2. Pressure force on CA = $(P + \frac{\delta P}{\delta z} \Delta z) \times \Delta A \uparrow$

3. Weight of fluid element = $\rho g = (\Delta A \times \Delta z \times \rho \times g)$

4. Pressure force on surfaces BC & AD are equal & opposite.
For equilibrium of fluid element, we have

$$P \Delta A - (P + \frac{\delta P}{\delta z} \Delta z) \Delta A + \Delta A \times \Delta z \times \rho \times g = 0$$

$$\cancel{P \Delta A} - \cancel{P \Delta A} - \frac{\delta P}{\delta z} \Delta z \Delta A + \Delta A \Delta z \rho g = 0$$

$$\frac{\delta P}{\delta z} \Delta z \Delta A = \Delta A \Delta z \rho g$$

$$\Rightarrow \boxed{\frac{\delta P}{\delta z} = \rho g} \quad (\because \rho g = w) \text{ weight density}$$

$$\Rightarrow \boxed{\text{Hydrostatic Law}}$$

By Integrating the above equation

$$\int dP = \int \rho g dz$$

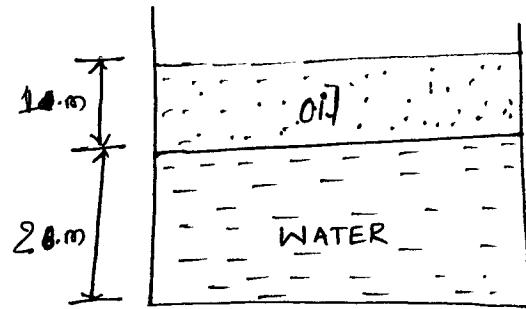
$$\boxed{P = \rho g z},$$

$\left. \begin{array}{l} \\ \end{array} \right\} z = \text{Pressure head}$

*Numericals:-

1. An open tank contains water upto a depth of 2 m & above it an oil of sp.gr. 0.9 for a depth of 1 m. Find the pres intensity (i) at the interface of two liquids (ii) at the bottom of the tank

Solve:-



Given that

$$\rho_1 = 1000 \text{ kg/m}^3 \quad \& \quad \rho_2 = 900 \text{ kg/m}^3$$

$$g = 9.81 \text{ m/s}^2$$

(i) Press intensity at interface

$$P = \rho g z$$

$$P = 900 \times 9.81 \times 1$$

$$\Rightarrow P = 8829 \text{ N/m}^2$$

(ii) Pressure intensity at Bottom

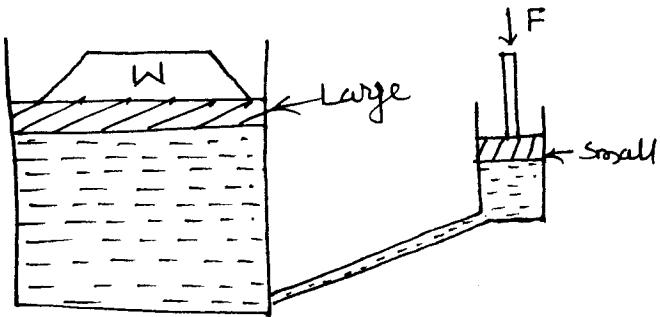
$$P = \rho_1 g z_1 + \rho_2 g z_2$$

$$P = 1000 \times 9.81 \times 2 + 900 \times 9.81 \times 1$$

$$\Rightarrow P = 28449 \text{ N/m}^2$$

2. The diameters of a small piston & a large piston of a hydraulic jack are 3 cm & 10 cm respectively. A force of 80N is applied on the small piston. Find the load lifted by the large piston when:
- The pistons are at the same level
 - Small piston is 40cm above the large piston.

Solution :-



Given That :-

$$\text{Dia of small piston} = 3 \text{ cm}$$

$$\text{Area} = \frac{\pi}{4} d^2 = \frac{\pi}{4} (3)^2 = 7.068 \text{ cm}^2$$

$$\text{Dia of large piston} \Rightarrow D = 10 \text{ cm.}$$

$$A = \frac{\pi}{4} (10)^2 = 78.54 \text{ cm}^2$$

$$F = 80 \text{ N}$$

(a) The pistons are at the same level

$$\text{Pressure on small piston} = P = \frac{F}{A} = \frac{80}{7.068} = 11.31 \text{ N/cm}^2$$

$$\therefore \text{Force on large piston} = P \times A$$

$$F = \frac{80}{7.068} \times 78.54 = 888.96 \text{ N}$$

(b) When small piston is 40 cm above the large

$$\begin{aligned} \therefore \text{Press intensity} &= \frac{F}{A} + \text{Press intensity due to height of } 40 \text{ cm of liquid} \\ &= \frac{F}{A} + \rho g h \\ &= \frac{80}{7.068} + \frac{1000 \times 9.81 \times 0.4}{10^4} \text{ N/cm}^2 \\ &= 11.32 + 0.3924 = 11.71 \text{ N/cm}^2 \end{aligned}$$

$$\therefore \text{force on large piston} = \text{Press} \times \text{Area} = 11.71 \times 78.54$$

$$F = 919.7 \text{ N}$$

* Pressure *

The pressure on a fluid is measured in two different systems. In one system, it is measured above the absolute zero or complete vacuum & it is called the absolute pressure & in other system, pressure is measured above the atmospheric pressure & it is called gauge pressure.

1. Absolute pressure :- It is defined as the pressure which is measured with reference to absolute zero or vacuum press.

2. Gauge Pressure :- It is defined as the pressure which is measured with the help of a pressure measuring instrument in which the atmospheric pressure is taken as datum. The atm. press on the scale is marked as zero.

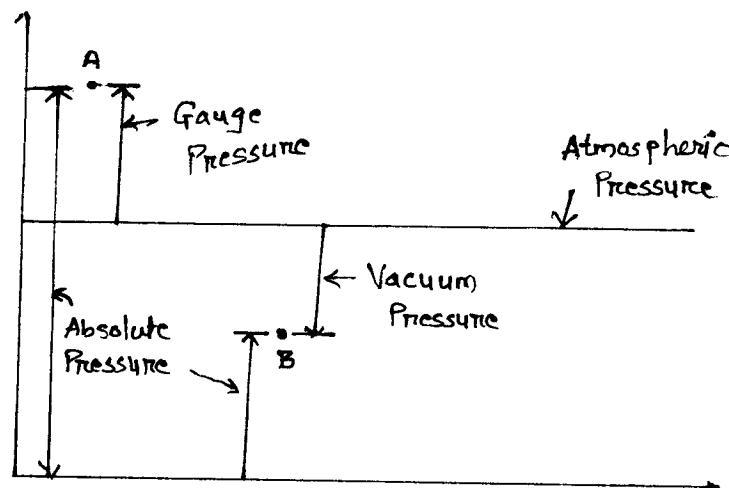
3. Vacuum pressure :- It is defined as the pressure below the atm pressure.

The Relationship b/w the absolute pressure, gauge press & vacuum pressure are shown

Mathematically

$$1. P_{ab} = P_{atm} + P_{gauge}$$

$$2. Vac(P) = P_{atm} - P_{abs}$$



* Measurement of pressure :-

The pressure of a fluid is measured by the following device

1. Manometer
2. Mechanical Gauges.

1. Manometer :- Manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of the fluid. They are classified as:

(a) Simple Manometers

(b) Differential Manometers.

* Mechanical Gauges:- Mechanical Gauges are defined as the devices used for measuring the pressure by balancing the fluid column by the spring or dead weight. The commonly used mechanical pressure gauges are:

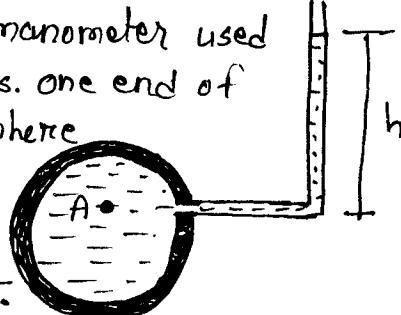
- (a) Diaphragm pressure gauge (b) Bourdon tube press gauge
- (c) Dead-weight pressure gauge (d) Bellows press gauge

* Simple Manometers:- A simple manometer consists of a glass tube having one of its ends connected to a point where pressure is to be measured & other end remains open to atmosphere. Common types of simple manometers are:

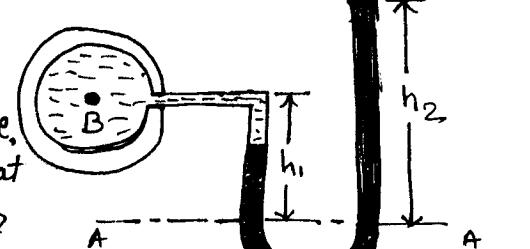
1. Piezometer 2. U-Tube manometer 3. Single column manometer

1. Piezometer:- It is the simplest form of manometer used for measuring gauge pressures. One end of this manometer is connected to the point where pressure is to be measured & other end is to be open to the atmosphere. The rise of liquid gives the pressure head at that point. If at a point A, the height of liquid say water is (h) in piezometer tube, then pressure at A

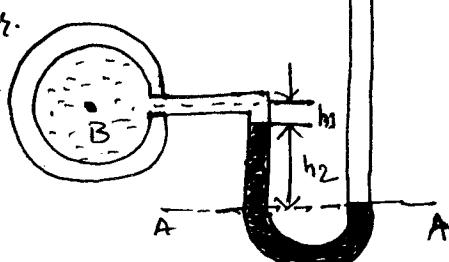
$$P = \rho g h \text{ N/m}^2$$



2. U-Tube Manometer:- It consists of glass tube bent in U-shape, one end of which is connected to a point at which pressure is to be measured & other end remains open to the atmosphere. The tube generally contains mercury or any other liquid whose sp. gr. is greater than the sp. gr. of the liquid whose pressure is to be measured.



a) for gauge pressure



(b) for Vacuum Pressure

(a) for Gauge pressure: Let B be the point at which pressure is to be measured, whose value is P. The datum line is A-A.

h_1 = Height of light liquid above the datum line

h_2 = Height of heavy lig. above the datum line

As the pressure is the same for the horizontal surface. Hence press above the horizontal datum line A-A in the left column & it the right column of U-tube manometer should be same.

$$\therefore P + \rho_1 g h_1 = \rho_2 g h_2$$

$$\therefore P = \rho_2 g h_2 - \rho_1 g h_1$$

(b) Gauge press

(b) for vacuum pressure:- For measuring vacuum pressure, the level of the heavy liquid in the manometer will be shown in fig 1(b). Then

$$\rho_2 g h_2 + \rho_1 g h_1 + P = 0$$

$$P = -(\rho_1 g h_1 + \rho_2 g h_2)$$

* Problem 1:- A U-Tube manometer is used to measure the press of water in a pipe line, which is in excess of atmospheric press. The right limb of the manometer contains mercury & is open to atmos. The contact b/w water & Hg is in the left limb. Determine the pressure of water in the main line, if the difference in level of mercury in the limbs of U-tube is 10 cm & the free surface of mercury is in level with the centre of the pipe. If the pressure of water in pipe line is reduced to 9810 N/m^2 , calculate the new difference in the level of mercury. Sketch the arrangement in both cases.

Solution:- Given:

$$\text{Difference of mercury} = 10 \text{ cm} = 0.1 \text{ m}$$

* Part A :-

Let pressure of the water in

$$\text{Pipe Line} = P_A \text{ (i.e at A)}$$

At point B →

$$= P_A + \rho g h.$$

$$= P_A + 1000 \times 9.81 \times 0.1$$

$$= P_A + 981 \text{ N/m}^2$$

At point C →

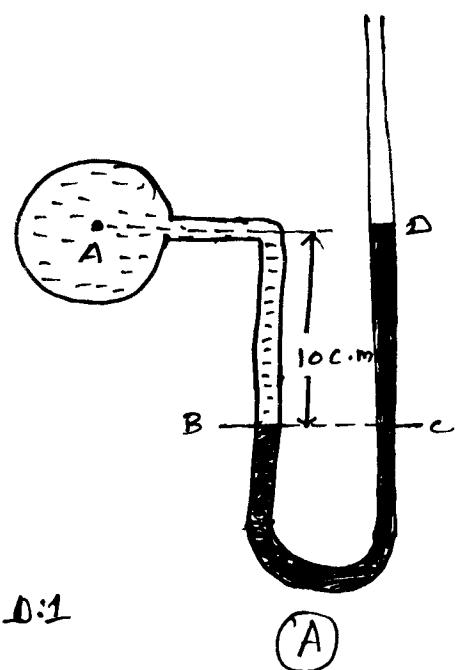
$$= \rho g h = 13.6 \times 1000 \times 9.81 \times 0.1$$

$$= 13341.6 \text{ N/m}^2$$

At equilibrium

$$P_A + 981 = 13341.6$$

$$\boxed{P_A = 12360.6 \text{ N/m}^2}$$



(A)

* Part B :-

At point B →

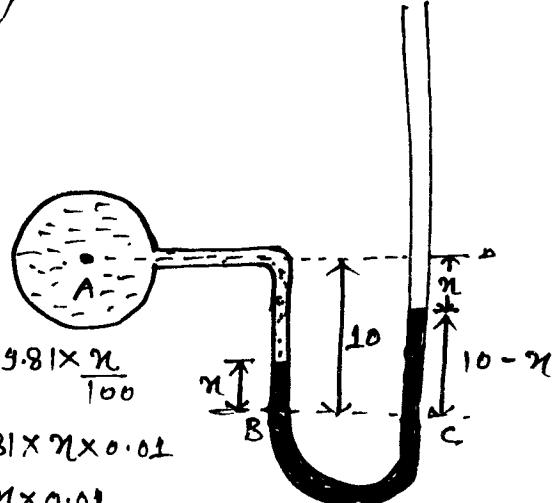
$$\Rightarrow P_A + \rho_1 g h_1 + \rho_2 g h_2$$

$$= 9810 + 1000 \times 9.81 \times \frac{(10-n)}{100} + 13600 \times 9.81 \times \frac{n}{100}$$

$$= 9810 + 981(0.1 - \frac{n}{100}) + 13600 \times 9.81 \times n \times 0.01$$

$$= 9810 + 98.1 - 981n + 133416n \times 0.01$$

$$\approx 132435n + 9908.1$$



At point C ⇒

$$= 13600 \times 9.81 \times \frac{(10-n)}{100}$$

$$= 133416 \frac{(10-n)}{100}$$

$$= 133416(0.1 - 0.01n)$$

$$= 13341.6 - 1334.16n$$

$$132435n + 9908.1 = 13341.6 - 1334.16n$$

$$2658.51n = 3433.5$$

$$n = 1.29 \text{ cm}$$

$$\therefore \Delta h = 10 - 2n = 7.42 \text{ cm} \text{ Ans.}$$

* Single column Manometer :-

Single column Manometer is a modified form of a U-tube manometer in which a reservoir, having a large cross-sectional area as compared to the area of the tube is connected to one of the limbs of the manometer as shown in fig. Due to large cross-sectional area of reservoir, for any variation in pressure, the change in the liquid level in the reservoir will be very small which may be neglected & hence the pressure is given by the height of liquid in the other limb. The other limb may be vertical or inclined. Thus there are two types of single column manometer as:

1. Vertical single column Manometer.
2. Inclined single column Manometer.

1. Vertical single column Manometer :-

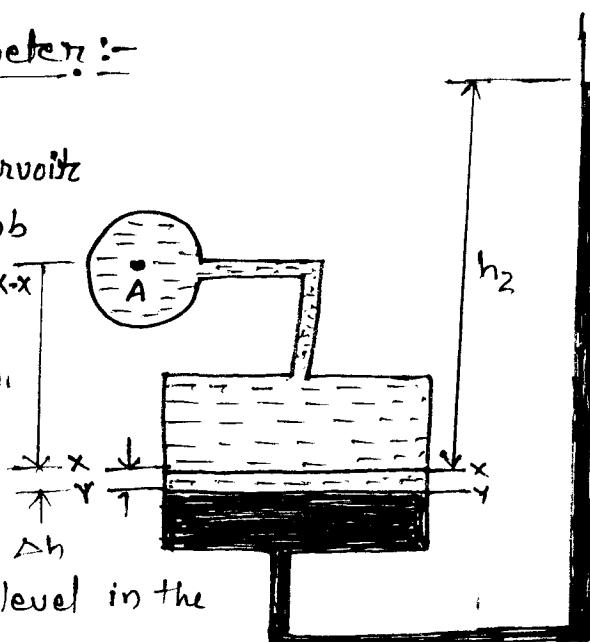
Δh = Fall of heavy liquid in reservoir

h_2 = Rise of heavy lig in right limb

h_1 = Height of centre of pipe above x-x

A = Cross-sectional area of reservoir

a = Cross-sectional area of the right limb



Fall of heavy liquid in reservoir
will cause a rise of heavy liquid level in the right limb.

$$\therefore A \times \Delta h = a \times h_2$$

$$\Delta h = \frac{a \times h_2}{A}$$

Now consider the datum line y-y as shown in fig. Then pressure in the right limb above y-y.

$$= P_2 \times g \times (\Delta h + h_2)$$

Pressure in left limb above y-y = $P_1 \times g \times (\Delta h + h_1) + P_A$

$$\therefore P_2 g (\Delta h + h_2) = P_1 g (\Delta h + h_1) + P_A$$

$$P_A = \frac{a}{A} \times h_2 [P_2 g - P_1 g] + h_2 P_2 g - h_1 P_1 g$$

$A \gg a \therefore a/A$ neglected

$$\boxed{\Delta h = \frac{a}{A} h_2}$$

$$\boxed{P_A = h_2 P_2 g - h_1 P_1 g}$$

2. Inclined Single Column Manometer:-

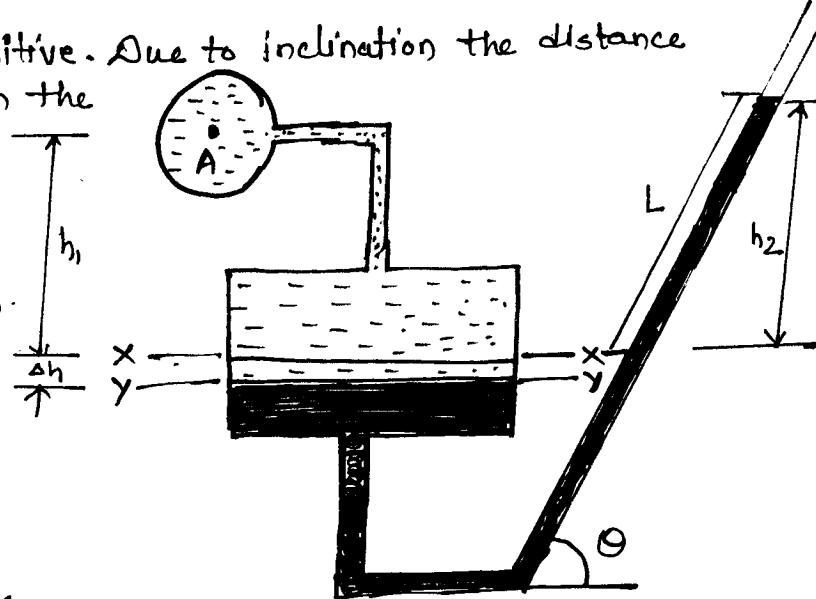
This manometer is more sensitive. Due to inclination the distance moved by the heavy liquid in the right limb will be more.

Let \Rightarrow

L = Length of heavy liquid moved in right limb from $X-X$

θ \Rightarrow Inclination of right limb with horizontal

$$h_2 = L \sin \theta$$



$$\therefore P_A + h_1 g = h_2 g$$

$$P_A = h_2 g - h_1 g$$

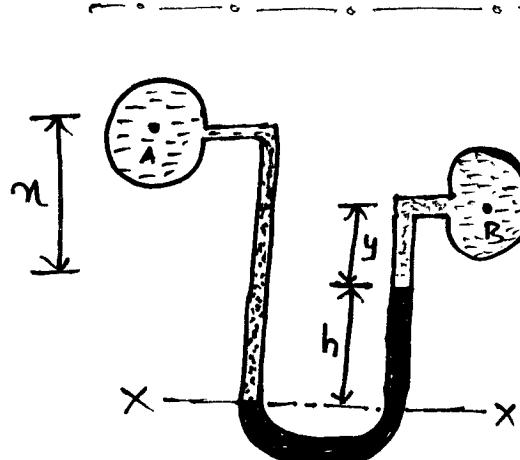
$$P_A = L \sin \theta g - h_1 g$$

* Differential Manometers:-

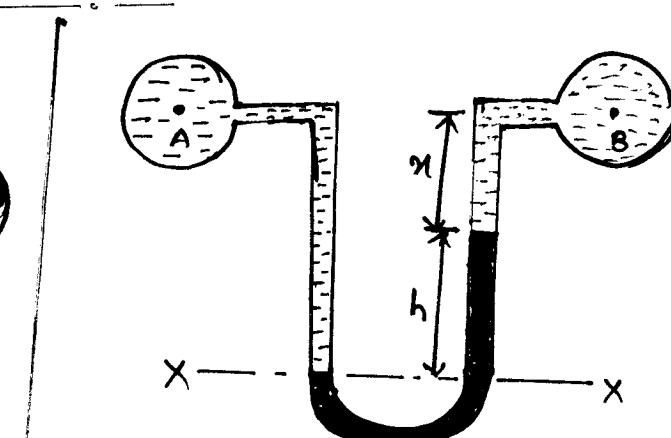
Differential manometers are the devices used for measuring the difference of pressures b/w two points in a pipe or in two different pipes. A differential manometer consists of a U-tube, containing a heavy liquid, whose two ends are connected to the points, whose difference of pressure is to be measured. Most commonly types of differential manometers are:

1. U-Tube differential manometer.
2. Inverted U-tube manometer.

1. U-tube differential manometer:-



(a) Two pipes at diff levels



(b) A & B are at the same level

(a) Two pipes at different levels :-

$$\rho_1 gh(h+n) + P_A = \rho_m gh + \rho_2 gn + P_B$$

$$P_A - P_B = \rho_m gh + \rho_2 gn - \rho_1 gh - \rho_2 gn$$

$$P_A - P_B = \rho_m gh + \rho_2 gn - \rho_1 gh - \rho_1 gn$$

(b) A & B are at the same level :- It contains same liquid of density ρ_1

$$\rho_m gh + \rho_1 gn + P_B = \rho_1 g(h+n) + P_A$$

$$P_A - P_B = \rho_m gh + \rho_1 gn - \rho_1 gh - \rho_1 gn$$

$$P_A - P_B = \rho_m gh - \rho_1 gh$$

$$\Rightarrow P_A - P_B = g \times h (\rho_m - \rho_1)$$

* Inverted U-tube differential Manometer :-

It is used for measuring difference of low pressures.

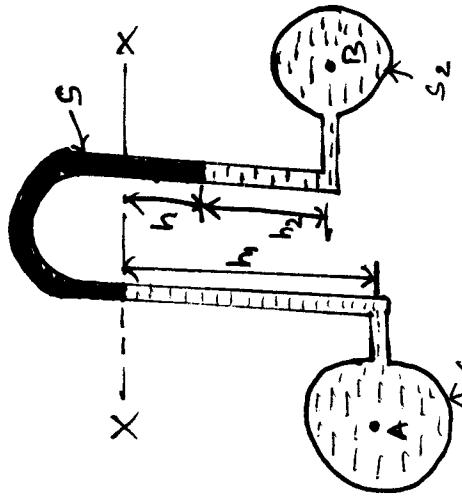
Let the pressure at A is more than the pressure at B

$$\Rightarrow P_A - P_1 \propto g \times h_1$$

$$\Rightarrow P_B - \rho_2 gh_2 - \rho_s gn$$

$$\therefore P_A - P_1 \rho_1 gh_1 = P_B - \rho_2 gh_2 - \rho_s gn$$

$$\Rightarrow P_A - P_B = \rho_1 gh_1 - \rho_2 gh_2 - \rho_s gn$$



* METHODS OF DIMENSIONAL ANALYSIS

If the number of variables involved in a physical phenomenon are known, then the relation among the variables can be determined by the following two methods:-

1. Rayleigh's Method
2. Buckingham's π - theorem

1. Rayleigh's Method: This Method is used for determining the expression for a variable which depends upon maximum three or four variables only. If number of independent variables becomes more than four, then it is very difficult to find the expression for the dependent variable.

Let X is a variable, which depends on x_1, x_2, x_3 variables. Then according to Rayleigh's method, X is function of $x_1, x_2 \& x_3$ & Mathematically it is written as

$$X = f[x_1, x_2, x_3]$$

This can also be written as

$$X = K x_1^a \cdot x_2^b \cdot x_3^c$$

where K is constant & a, b, c are arbitrary powers.

The values of $a, b \& c$ are obtained by comparing the powers of the fundamental dimension on both sides. Thus the expression is obtained for dependent variable.

Problem 1:- The time period (t) of a pendulum depends upon the length (L) of the pendulum & acceleration due to gravity (g). Write an expression for the time period.

Solution:- Time period t is a function of (i) L (ii) g

$$t = K L^a g^b$$

Substituting dimensions on both sides

$$T = K L^a (L T^{-2})^b \quad \text{--- (1)}$$

$$\therefore \text{Power of } T \quad 1 = -2b$$

$$\therefore b = -\frac{1}{2}$$

Power of L

$$0 = a + b$$

$$a = \frac{1}{2}$$

$$\therefore t = K L^{1/2} g^{-1/2}$$

$$t = K \sqrt{L/g}$$

$K = 2\pi$ from experiment

$$t = 2\pi \sqrt{\frac{L}{g}}$$

* Buckingham's Π -Theorem:— The Rayleigh's method of dimensional analysis becomes more laborious if the variables are more than the number of fundamental dimensions (M, L, T). This difficulty is overcome by using Buckingham's Π -theorem, which states "If there are n variables (independent & dependent) in a physical phenomenon & if these variables contain ' m ' fundamental dimensions (M, L, T), then the variables are arranged into $(n-m)$ dimensionless terms. Each term is called Π -term."

Let $x_1, x_2, x_3, \dots, x_n$ are the variables involved in a physical problem. Let x_1 be the dependent variable & x_2, x_3, \dots, x_n are the independent variables on which x_1 depends. Then x_1 is a function of x_2, x_3, \dots, x_n & mathematically it is expressed as

$$x_1 = f(x_2, x_3, \dots, x_n) \quad \text{--- (1)}$$

Eq(1) also can be written as

$$f(x_1, x_2, x_3, \dots, x_n) = 0 \quad \text{--- (2)}$$

Equation (2) is a dimensionally homogeneous equation. It contains n variables. If there are m fundamental dimension then according to Buckingham's Π -theorem, eq(2) can be

written in terms of number of dimensionless group of π -terms is equal to $(n-m)$. Hence eqn (2) becomes as

$$f(\pi_1, \pi_2, \dots, \pi_{n-m}) = 0 \quad \text{--- (3)}$$

Each of π -term is dimensionless & is independent of the system. Division or multiplication by a constant does not change the character of the π -term. Each π -term contains $m+1$ variables where m is the number of fundamental dimensions & is also called repeating variables. Let in the above case x_2, x_3 & x_4 are repeating variables if the fundamental dimension in $(m, l, t) = 3$. Then each π -term is written as.

$$\left. \begin{aligned} \pi_1 &= x_2^{a_1} \cdot x_3^{b_1} \cdot x_4^{c_1} \cdot x_1 \\ \pi_2 &= x_2^{a_2} \cdot x_3^{b_2} \cdot x_4^{c_2} \cdot x_5 \\ &\vdots \\ \pi_{n-m} &= x_2^{a_{n-m}} \cdot x_3^{b_{n-m}} \cdot x_4^{c_{n-m}} \cdot x_n \end{aligned} \right\} \quad \text{--- (4)}$$

Each eqn is solved by the principle of dimensional homogeneity & values of a_1, b_1, c_1 etc. are obtained. These values are substituted in eqn (4) & values of $\pi_1, \pi_2, \dots, \pi_{n-m}$ are obtained. These values of π 's are substituted in eqn [3]. The final eqn for the phenomenon is obtained by expressing any one of the π -terms as a function of others as

$$\left. \begin{aligned} \pi_1 &= \phi[\pi_2, \pi_3, \dots, \pi_{n-m}] \\ \pi_2 &= \phi[\pi_1, \pi_3, \dots, \pi_{n-m}] \end{aligned} \right\}$$

* Method of Selecting Repeating Variables :-

The number of repeating variables are equal to the number of fundamental dimensions of the problem. The choice of the repeating variables is governed by the following considerations.

1. As far as possible, the dependent variable should not be selected as repeating variables.

2. The repeating variables should be chosen in such a way that one variable contains geometric property, other variable contains flow property & 3rd variable contains fluid property
Variables with Geometric property
(i) length(l) (ii) d (iii) Height H etc.

Variables with flow property are.

(i) Velocity v (ii) Acceleration a etc.

Variables with fluid property

(i) μ (ii) ρ (iii) w etc.

3. The repeating variables selected should not form a dimensionless group.

4. The repeating variables together must have the same number of fundamental dimensions.

5. No two repeating variables should have the same dimensions.

In most of fluid mechanics problems, the choice of repeating variables may be (i) d, v, s (ii) L, v, s (iii) L, v, μ (iv) d, v, μ

* Procedure for solving problems by Buckingham's π - theorem :-

Problem!:- The resisting force R of a supersonic plane during flight can be considered as dependent upon length of aircraft L, velocity v, air viscosity μ , air density ρ & bulk modulus of air K. Express the functional relationship b/w these variables & the resisting force.

Solution!:- Step I!:- R is a function of L, v, μ , ρ , K

$$\therefore R = f(L, v, \mu, \rho, K)$$

$$\text{or } f_1(L, v, \mu, \rho, K, R) = 0$$

Total no of variables n=6

No of fundamental dimensions, m=3

[m is obtained by writing dimensions of each variables as

$R = MLT^{-2}$, $v = LT^{-1}$, $\mu = ML^{-1}T^{-1}$, $\rho = ML^{-3}$, $K = ML^{-1}T^{-2}$. Thus as fundamental dimensions in the prob are M, L, T. $\therefore m=3$]

∴ Number of π -term = $n-m = 6-3 = 3$

$$\therefore f(\pi_1, \pi_2, \pi_3) = 0$$

Step 2:- Each π -term = $m+1$ variables,

out of 6 variables 3 should be repeating. R is dependent variable & should not be selected as repeating variable

∴ Repeating variables are V, L & S. The repeating variables themselves should not form a dimensionless term & should have themselves fundamental dimensions equal to m i.e 3 here

Step 3:- Each π -term is written as according to eqn

$$\left. \begin{array}{l} \pi_1 = L^{a_1}, V^{b_1}, S^{c_1}, R \\ \pi_2 = L^{a_2}, V^{b_2}, S^{c_2}, M \\ \pi_3 = L^{a_3}, V^{b_3}, S, K \end{array} \right\}$$

Step 4:- Each π -term is solved by the principle of dimensional homogeneity. For the 1st π -term, we have

$$\pi_1 = M^0 L^0 T^0 = L^{a_1} (LT^{-1})^{b_1} (ML^{-3})^{c_1} MLT^{-2}$$

$$0 = c_1 + 1 \quad \therefore c_1 = -1$$

$$0 = a_1 + b_1 - 3c_1 + 1$$

$$0 = -b_1 - 3 - 1 = -2$$

$$0 = -b_1 - 2 = b_1 = -2$$

$$\therefore \pi_1 = L^{-2} V^{-2} S^{-1} R$$

$$\therefore \pi_1 = \frac{R}{S L^2 V^2} \quad \text{--- (1)}$$

Similarly

$$\pi_2 = M^0 L^0 T^0 = L^{a_2} (LT^{-1})^{b_2} (ML^{-3})^{c_2} ML^2 T^{-1}$$

$$0 = c_2 + 1 \quad \therefore c_2 = -1$$

$$0 = a_2 + b_2 - 3c_2 - 1 \quad \therefore a_2 = -1$$

$$0 = -b_2 - 1 \quad \therefore b_2 = -1$$

$$\pi_2 = L^1 V^{-1} S^{-1} \cdot M = \frac{M}{L V S}$$

Similarly \rightarrow

$$\pi_3 = M^0 L^0 T^0 = L^{a_3} \cdot (LT^{-2})^{b_3} (ML^{-3})^{c_3} ML^{-1} T^{-2}$$

$$\therefore 0 = c_3 + 1 \therefore c_3 = -1$$

$$0 = a_3 + b_3 - 3c_3 - 1 \therefore a_3 = 0$$

$$0 = -b_3 - 2 \therefore b_3 = -2$$

$$\pi_3 = L^0 \cdot V^{-2} S^{-1} K = \frac{K}{V^2 S}$$

Step 5:- $f\left(\frac{R}{SL^2 V^2}, \frac{M}{LVS}, \frac{K}{V^2 S}\right) = 0$

$$\frac{R}{SL^2 V^2} = \phi \left[\frac{M}{LVS}, \frac{K}{V^2 S} \right]$$

$$\Rightarrow R = SL^2 V^2 \phi \left[\frac{M}{LVS}, \frac{K}{V^2 S} \right]$$

* Model Analysis:-

For predicting the performance of the hydraulic structures (such as dams, spillways etc) or hydraulic machine (such as turbine, pumps etc), before actually constructing or manufacturing, models of the structures or machines are made & tests are performed on them to obtain the desired information.

The model is the small scale replica of the actual structure or machine. The actual structure or machine is called prototype. It is not necessary that the models should be smaller than the prototypes, they may be larger than the prototype. The study of models of actual machines is called Model analysis. Model analysis is actually an experimental method of finding solutions of complex flow problems.

* Dimensionless Numbers :-

1. Reynold's Number (R_e) \rightarrow It is defined as the ratio of inertia force of a flowing fluid & viscous force of the fluid.

$$\begin{aligned} \text{Inertia force } F_i &= M \times a \\ &= \rho \times V \times \frac{\cancel{A}}{\cancel{t}} \quad \left. \begin{array}{l} \text{Volume per sec} \\ = V \times A \end{array} \right\} \\ &= \rho \times A \times \cancel{V} \times \frac{\cancel{V}}{\cancel{t}} \\ &= \rho \times A \times V^2 \end{aligned}$$

Viscous force (F_v) = shear stress \times Area

$$\begin{aligned} &= \tau \times A \\ &= \mu \frac{du}{dy} \times A = \mu \frac{V}{L} \times A \end{aligned}$$

$$\therefore R_e = \frac{S A V^2}{\mu \frac{V}{L} A} = \frac{\rho g L}{\mu}$$

for pipe \rightarrow
$$R_e = \frac{d u_s}{\mu}$$
, d - diameter of pipe

2. Froude's Number (F_f):- It is defined as the ratio of the inertia force to the gravity force.

$$\begin{aligned} F_f &= \sqrt{\frac{F_i}{F_g}} = \sqrt{\frac{\rho A V^2}{\rho A L g}} \\ &\boxed{F_f = \left[\frac{V}{\sqrt{L g}} \right]} \end{aligned}$$

3. Euler's Number: $E_u \Rightarrow$ Ratio of pressure force to inertia force

$$\Rightarrow \boxed{E_u = \frac{P}{\rho V^2}}$$

* KINEMATICS OF FLOW *

Kinematics is defined as that branch of science which deals with motion of particles without considering the forces causing the motion. The velocity at any point in flow field at any time is studied in this branch of fluid mechanics. Once the velocity is known then the pressure distribution & hence force acting on the fluid can be determined.

* Methods of describing fluid Motion :-

The fluid motion is described by two methods

(i) Lagrangian Method.

(ii) Eulerian Method.

In 1st method a single fluid particle is followed during its motion & its velocity, acceleration, density etc. are described.

In case of Eulerian method, the velocity, acceleration, press density etc. are described at a point in flow field. The Eulerian method is commonly used in fluid mechanics.

* Types of fluid flow:-

1. Steady & unsteady flow:- Steady flow is defined as that type of flow in which the fluid characteristics like velocity, pressure, density, etc. at a point do not change with time.

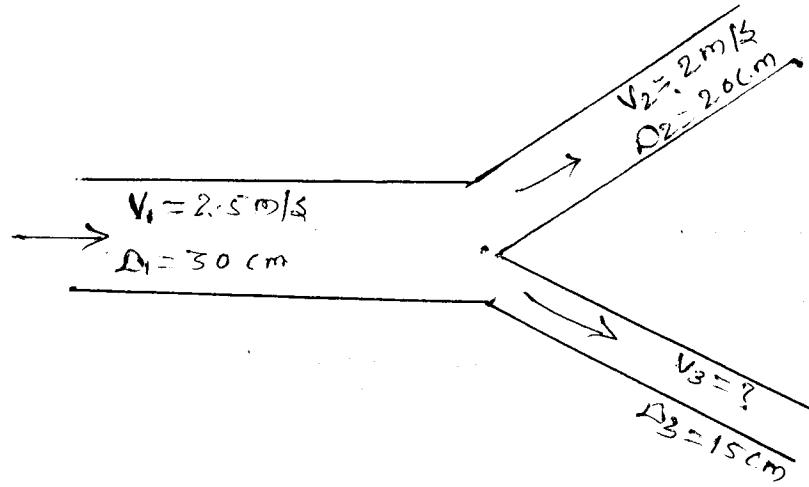
$$\left(\frac{\delta V}{\delta t}\right)_{x_0, y_0, z_0} = 0, \quad \left(\frac{\delta P}{\delta t}\right)_{x_0, y_0, z_0} = 0, \quad \left(\frac{\delta \rho}{\delta t}\right)_{x_0, y_0, z_0} = 0$$

And for unsteady flow all parameters changes with respect to time.

2 Uniform & Nonuniform flow:- Uniform flow is defined as that type of flow in which the velocity at any given time does not change with respect to space (i.e length of direction of the flow). Mathematically

$$\left(\frac{\delta V}{\delta S}\right)_{t-\text{cons}} = 0 \quad \begin{cases} \text{for Non uniform flow} \\ \left(\frac{\delta V}{\delta S}\right)_{t-\text{cons}} \neq 0 \end{cases}$$

Solution:-



$$A_1 = \frac{\pi}{4} (d^2) = \frac{\pi}{4} \times (30 \times 10^{-2})^2$$

$$A_1 = 7.064 \times 10^{-2} \text{ m}^2$$

$$A_2 = 3.14 \times 10^{-2} \text{ m}^2$$

$$A_3 = 1.76 \times 10^{-2} \text{ m}^2$$

$$\therefore Q_1 = 0.1767 \text{ m}^3/\text{s}$$

$$\therefore Q_1 = Q_2 + Q_3$$

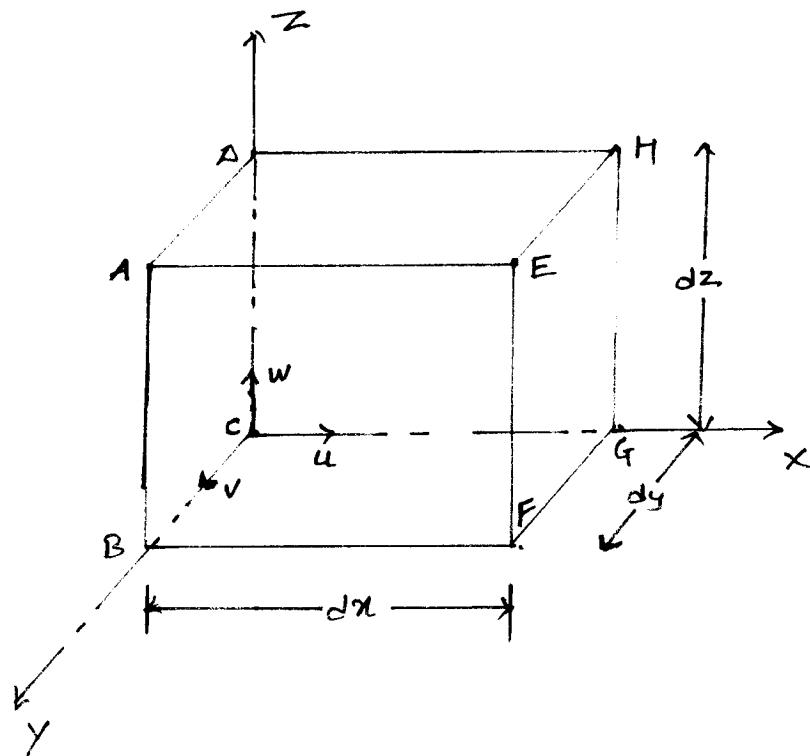
$$\therefore Q_2 = 0.0628$$

$$\therefore Q_3 = 0.1139 \text{ m}^3/\text{s}$$

$$\therefore V_3 = \frac{Q_3}{A_3} = 6.44 \text{ m/s}$$

$V_3 = 6.44 \text{ m/s}$

* Continuity Equation in Three-dimensions:-



Consider a fluid element of length $dx, dy \& dz$ in the direction of x, y, z . Let $u, v \& w$ are the inlet velocity components in x, y, z directions respectively. Mass of fluid entering the face ABCD per second.

$$= \rho \times \text{velocity in } x\text{-direction} \times \text{area of ABCD}$$

$$= \rho \times u \times (dy \times dz)$$

Then mass of fluid leaving the face EFGH per second

$$= \rho u dy dz + \frac{\delta}{\delta n} (\rho u dy dz) dn$$

\therefore Gain of mass in x -direction

$$= \text{mass through ABCD} - \text{mass through EFGH}$$

$$= \cancel{\rho u dy dz} - \cancel{\rho u dy dz} - \frac{\delta}{\delta n} (\rho u dy dz) dn$$

$$= -\frac{\delta}{\delta n} (\rho u dy dz) dn$$

$$= -\frac{\delta}{\delta n} (\rho u) dn dy dz$$

Similarly, the net gain of mass in y -direction

$$= -\frac{\delta}{\delta y} (\rho v) dn dy dz$$

$$z\text{-direction} = -\frac{\delta}{\delta z} (\rho w) dn dy dz$$

$$\therefore \text{Net gain of masses} = - \left[\frac{\delta}{\delta t} (\rho u) + \frac{\delta}{\delta y} (\rho v) + \frac{\delta}{\delta z} (\rho w) \right] dxdydz$$

~~Since~~ since the mass is neither be created nor be destroyed in the fluid element, the net increase of mass per unit time in the fluid element must be equal to the rate of increase of mass of fluid in the element. But mass of fluid in the element is $\rho dx dy dz$ & its rate of increase with time is $\frac{\delta \rho}{\delta t} (dx dy dz)$ or $\frac{\delta \rho}{\delta t} dx dy dz$.

Equating the two expressions.

$$- \left[\frac{\delta}{\delta t} (\rho u) + \frac{\delta}{\delta y} (\rho v) + \frac{\delta}{\delta z} (\rho w) \right] dxdydz = \frac{\delta \rho}{\delta t} dx dy dz$$

$$\frac{\delta \rho}{\delta t} + \frac{\delta}{\delta x} (\rho u) + \frac{\delta}{\delta y} (\rho v) + \frac{\delta}{\delta z} (\rho w) = 0$$

This equation is applicable to

- (i) Steady & unsteady flow
- (2.) Uniform & Non uniform flow
- (3.) Compressible & incompressible fluids

\therefore for steady flow $\frac{\delta \rho}{\delta t} = 0$

$$\therefore \frac{\delta}{\delta x} (\rho u) + \frac{\delta}{\delta y} (\rho v) + \frac{\delta}{\delta z} (\rho w) = 0 \quad \text{--- (1)}$$

for in-compressible fluid

$$\frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} + \frac{\delta w}{\delta z} = 0 \quad \text{--- (2)}$$

For two-dimensional flow

$$\frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} = 0 \quad \text{--- (3)}$$

* Velocity potential function & stream function

* Velocity potential function :- It is defined as a scalar function of space & time such that its (-)ve derivative with respect to any direction gives the fluid velocity in that direction. It is defined by ϕ . Mathematically

$$\phi = f(x, y, z) \text{ for steady flow}$$

$$\left\{ \begin{array}{l} u = -\frac{\partial \phi}{\partial x} \\ v = -\frac{\partial \phi}{\partial y} \\ w = -\frac{\partial \phi}{\partial z} \end{array} \right.$$

The velocity components in cylindrical polar co-ordinates in terms of velocity potential function are given by

$$u_r = \frac{\partial \phi}{\partial r}, \quad u_\theta = \frac{1}{r} \frac{\partial \phi}{\partial \theta}$$

u_r = Velocity component in radial direction (i.e r dirⁿ)

u_θ = Velocity component in tangential direction (i.e θ dirⁿ)

The continuity equation for an incompressible steady flow is

$$\frac{\partial u}{\partial r} + \frac{\partial v}{\partial \theta} + \frac{\partial w}{\partial z} = 0$$

Substituting the value of u, v, w

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{\partial^2 \phi}{\partial \theta^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \rightarrow \text{Laplace eqn}$$

Any value of ϕ that satisfies the Laplace eqn will correspond to some case of fluid flow.

* Properties of potential flow :- The rotational components are given by

$$w_z = \frac{1}{2} \left(\frac{\partial v}{\partial r} - \frac{\partial u}{\partial \theta} \right)$$

(1.) If velocity potential (ϕ) exists, the flow should be irrotational flow.
 This means $\omega = 0$ means irrotational flow.

$$\left(\frac{\partial \phi}{\partial y} - \frac{\partial \phi}{\partial z} \right) \frac{\partial}{\partial y} = \omega$$

$$\left(\frac{\partial \phi}{\partial z} - \frac{\partial \phi}{\partial y} \right) \frac{\partial}{\partial z} = \omega$$

The possible steady irrotational flow.

(2) If velocity potential (ϕ) exists, it represents

* Fluid flow phenomena *

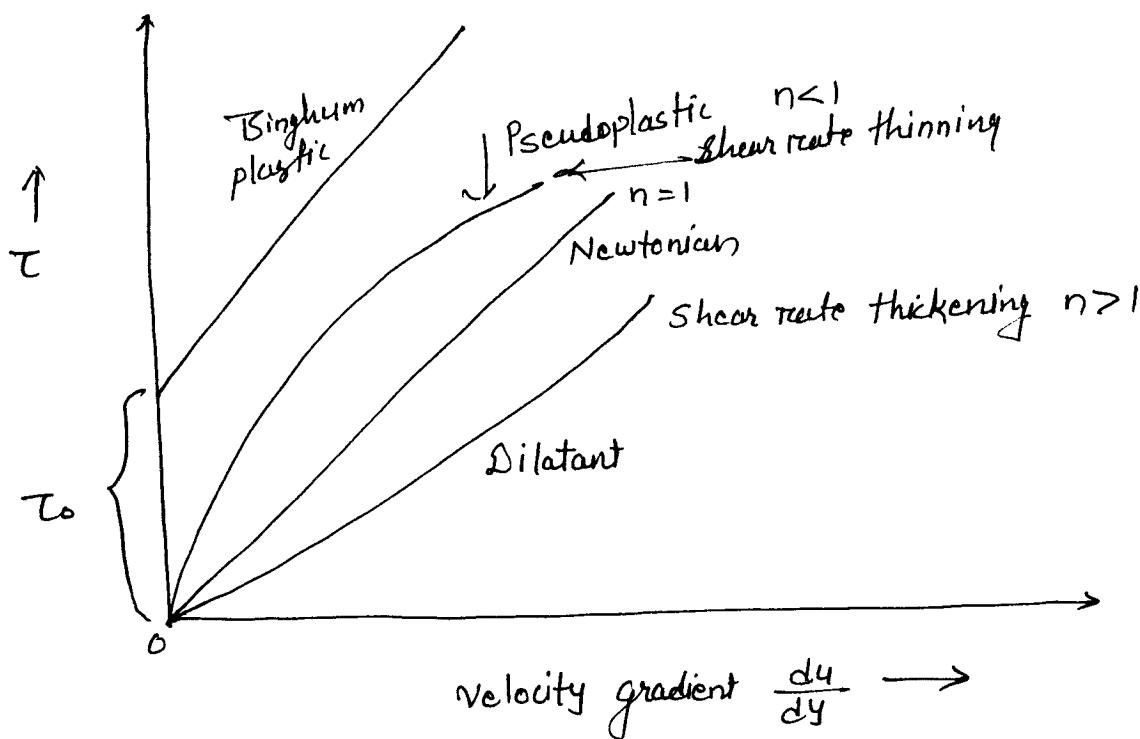
- * The behavior of a flowing fluid depends strongly on whether or not the fluid is under the influence of solid boundaries.
- * In the region where the influence of the wall is small, the shear stress may be negligible & the fluid behavior may approach that of an ideal fluid, one that is incompressible & has zero viscosity. The flow of such an ideal fluid, one that is incompressible & has zero viscosity. The flow of such an ideal fluid is called potential flow & is completely described by the principles of ~~Newtonian~~ Newtonian mechanics & conservation of mass.
- * Potential flow has two important characteristics
 - (i) Neither circulations nor eddies can form within the stream, so that potential flow is also called irrotational flow
 - (2.) Friction can not develop, so that there is no dissipation of mechanical energy into heat.
- * The effect of the solid boundary on the flow is confined to a layer of the fluid immediately adjacent to the solid wall. This layer is called the boundary layer, and shear & shear forces are confined to this part of the fluid. Outside the boundary layer potential flow survives.
- * In some situations such as flow in a converging nozzle, the boundary layer may be neglected & in others such as flow through pipes, the boundary layer fills the entire channel, & there is no potential flow.
- * The velocity field:- When a stream of fluid is flowing in bulk past a solid wall, the fluid adheres to the solid at the actual interface b/w solid & fluid. The adhesion is a result of the force fields at the boundary, which are also responsible for the interfacial tension b/w solid & fluid.
- * The velocity of the fluid at interface is zero {wall at rest}

* Shear stress :- The force per unit area of the shearing plane, called the shear stress & denoted by τ or

$$\boxed{\tau = \frac{F_s}{A_s}}, \quad A_s = \text{Area of plane}$$

* Shear forces are generated in both laminar & turbulent flow.

* Newtonian & Non-Newtonian fluid :- At constant T & P



τ_0 = Threshold shear stress

Shear stress

* Thixotropic :- Their apparent viscosity decreases with time

* Rheopectic :- Their apparent viscosity & shear stress increases with time.

* Newton's Law of viscosity :- In a newtonian fluid the shear stress is proportional to the shear rate & the proportionality constant is called the viscosity.

1

$$\boxed{\tau_n = \mu \frac{du}{dy}}$$

$$\mu \rightarrow \frac{Ns}{m^2} \rightarrow \frac{kg}{ms}$$

⇒ Momentum transfer is analogous to conductive heat transfer resulting from a temperature gradient,

* Viscosity of gases & liquid:-

Viscosity of a newtonian fluid depends primarily on temperature & to a lesser degree on pressure.

μ_{gas} ↑ with temperature

$$\frac{\mu}{\mu_0} = \left(\frac{T}{273} \right)^n$$

μ = viscosity at abs temp T K
 μ_0 = viscosity at 0°C (273K)
 n = constant.

at high pressure μ_{gas} ↑ → with pressure.

$$\mu_{\text{liq}} \downarrow \rightarrow \text{Temp} \uparrow$$

$$\mu_{\text{liq}} \uparrow \rightarrow P \uparrow$$

* Kinematic viscosity:- The ratio of the absolute viscosity to the density of a fluid.

$$\Rightarrow \boxed{\nu = \frac{\mu}{\rho}} \Rightarrow \frac{m^2}{s}$$

$$1 \text{ Stoke} = 1 \frac{cm^2}{s}$$

⇒ For liquids kinematic viscosity varies with temp over somewhat narrower range than absolute viscosity

⇒ For gases ν increases more rapidly with temperature than μ

* Rate of shear vs shear stress for Non-Newtonian fluids.

$$\boxed{\tau_v = \tau_0 + k \frac{du}{dy}} \rightarrow \text{for Bingham plastic}$$

Power law:-

$$\tau_v = k' \left(\frac{du}{dy} \right)^n$$

k' → flow consistency index
 n' → flow behavior index.

$n' < 1$ pseudoplastic

$n' > 1$ dilatant.

* Reynolds number \rightarrow

$$Re = \frac{d u s}{\mu}$$

$Re < 2100$ Laminar flow

$Re \text{ b/w } 2100 - 4000 \rightarrow \text{transition regime}$

$Re > 4000 \rightarrow \text{Turbulent}$

} Pipe flow

* Nature of Turbulence:-

(1) Wall Turbulence \rightarrow closed or open channels

(2) Free Turbulence \rightarrow Jet

* Flow within an eddy is laminar

* Turbulent flow is not a molecular phenomenon

* Intensity & scale of Turbulence:-

Turbulent fields are characterized by two avg. parameters. The first measure the intensity of the field & refers to the speed of rotation of the eddies and energy contained in an eddy of a specific size. The 2nd measures the size of the eddies. Intensity measured by the root mean square velocity component.

The scale of turbulence is based on correlation coefficient such as Ru' measured as a function of the distance b/w stations.

$$\therefore L_y = \int_0^\infty Ru' dy$$

* Isotropic Turbulence:- Root mean square components are equal for all directions at a given point.

In this situation the turbulence is said to be isotropic &

$$(\bar{u}')^2 = (\bar{v}')^2 = (\bar{w}')^2$$

Nearly isotropic turbulence exists when there is no velocity gradient, as the centreline of a pipe or beyond the outer edge of a boundary layer

* Reynolds stress :-

\Rightarrow Turbulent shear stress = Reynolds stress

* Eddy viscosity :-

$$T_t = E_v \frac{du}{dy}$$

E_v = Eddy viscosity

E_m = Eddy diffusivity of momentum

$$E_m = \frac{E_v}{S}$$

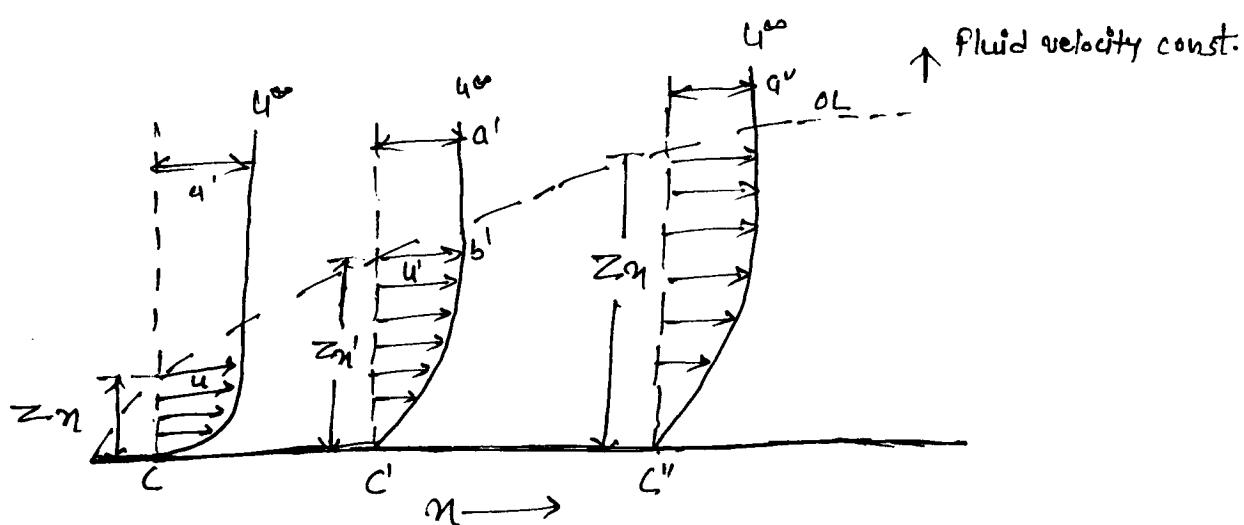
\therefore Total shear stress = τ

$$\tau = (\mu + E_v) \frac{du}{dy}$$

* The E_v & E_m are not just properties of the fluid but depends on the fluid velocity & the geometry of the system.

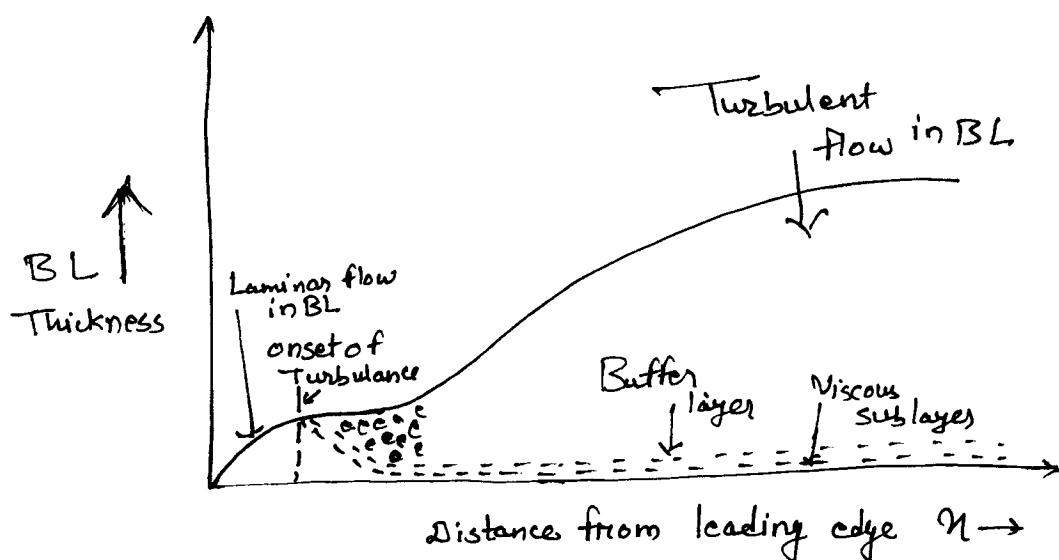
* Flow in Boundary layer :-

A boundary layer is defined as that part of a moving fluid in which the fluid motion is influenced by the presence of solid boundary.



$zeta_n$ = Thickness of bl

* Laminar & Turbulent flow in boundary layer:-



* Turbulent BL is considered to consist of 3 zones

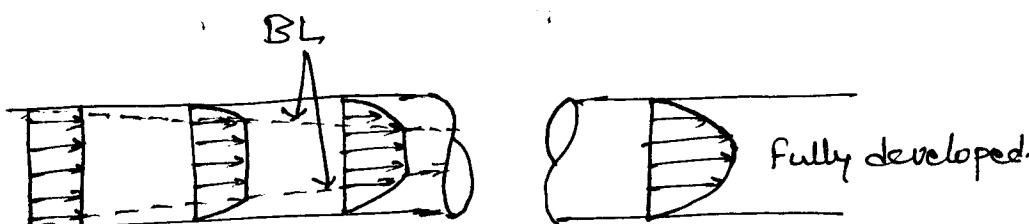
- (i) Viscous sublayer
- (ii) Buffer layer
- (iii) Turbulent layer (zone)

$z_n \uparrow$ with $n^{0.5}$ \rightarrow Laminar

$z_n \uparrow$ with $n^{1.5}$ \rightarrow for a short time after turbulence

$z_n \uparrow$ with $n^{0.8}$ \rightarrow Fully developed (Turbulent)

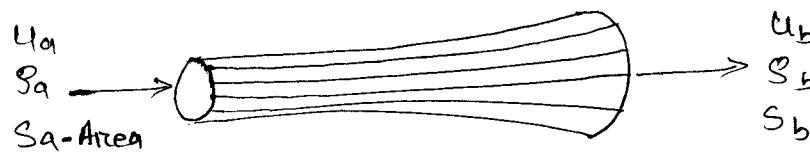
* BL formation in straight tubes:-



$$\frac{x_t}{D} = 0.05 Re \quad \text{Laminar flow}$$

x_t = transition length
 D = dia of pipe

* Average Velocity :-



The mass flow rate through differential areas in the cross section of a stream tube is

$$dm = \rho u dS$$

and the total mass flow rate through the entire cross-section

$$m = \rho \int_S u dS$$

The avg velocity \bar{V} of the entire stream flowing through cross sectional area S is defined as

$$\bar{V} = \frac{m}{\rho S} = \frac{1}{S} \int_S u dS$$

$$\boxed{\bar{V} = \frac{Q}{S}}$$

Q - Vol^m flow rate
 S - Area (cross-sectional)

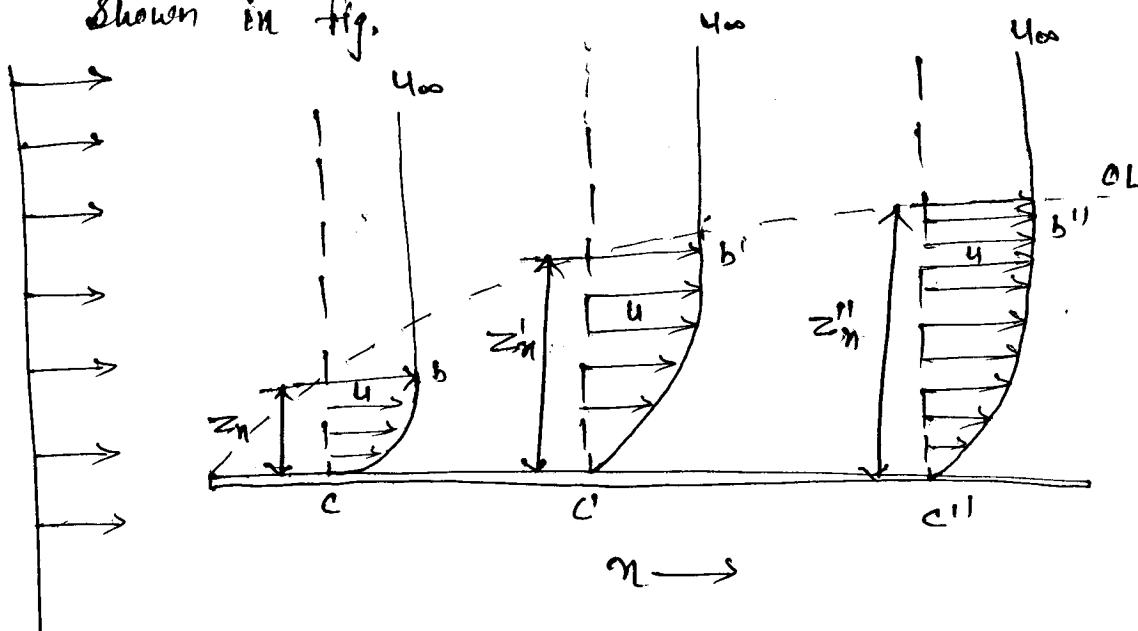
$$\text{Mass Velocity:- } \boxed{\frac{dm}{S} = \bar{V} S = G} \quad \frac{\text{kg}}{\text{sm}^2}$$

The advantage of using G is that it is independent of T & P when the flow is steady (cons(m)) & cross-sec is unchanged.

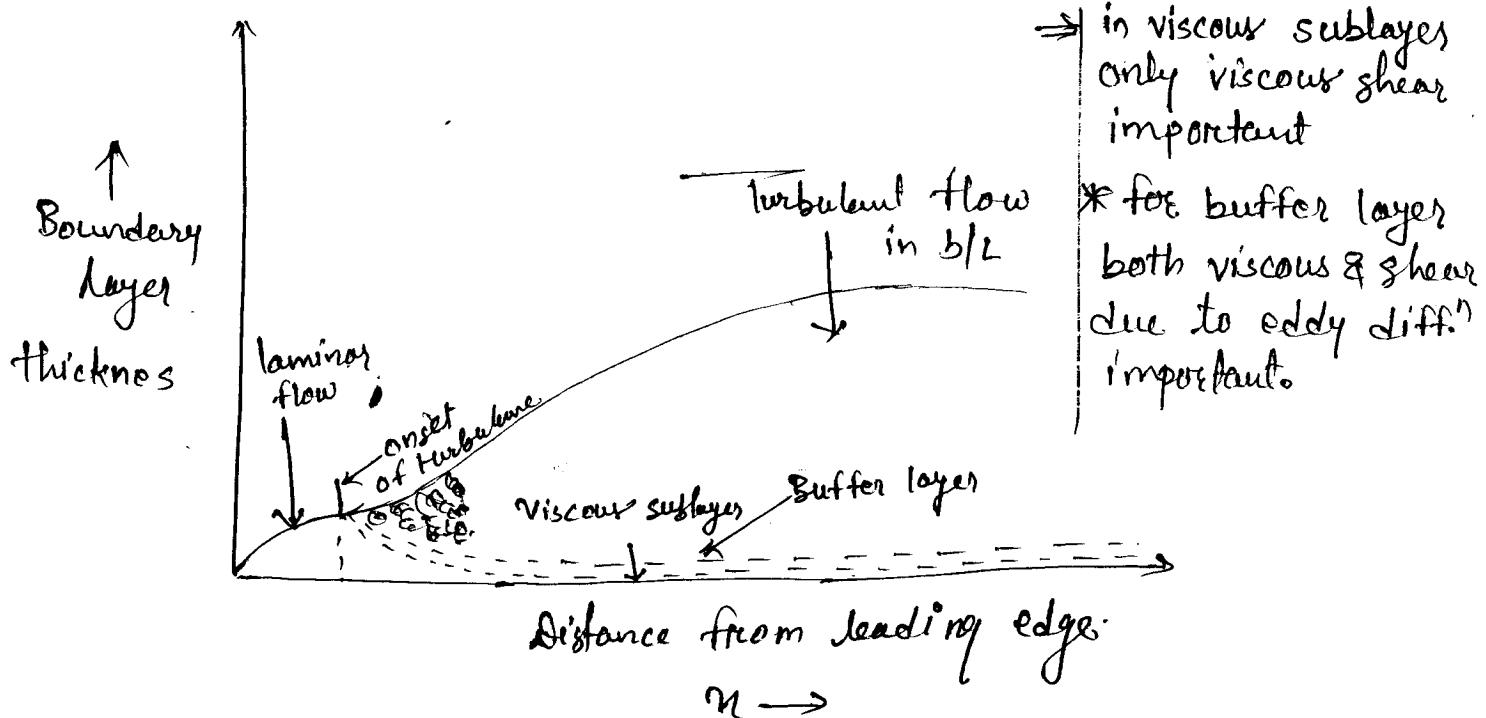
STATIONARY FLUID

* Flow in boundary layer:-

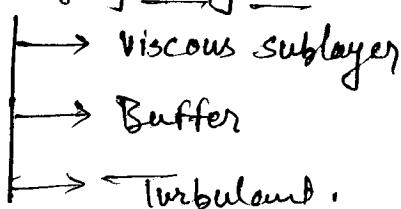
consider the flow of fluid parallel with a thin plate as shown in fig.



* Laminar & turbulent flow in boundary layers:-



Turbulent boundary layer:-



$$\begin{aligned}
 \delta_n &\rightarrow \text{thickness of BL} \\
 \text{laminar} &\rightarrow \delta_n = x^{0.5} \\
 \text{Turbulent appears} &\rightarrow \delta_n = x^{1.5} \\
 \text{fully developed} &\rightarrow \delta_n = x^{0.8}
 \end{aligned}$$

* Transition from laminar to turbulent flow

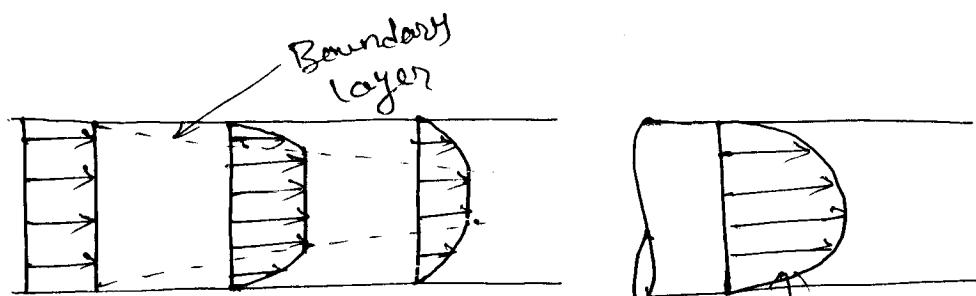
$$N_{Re} = \frac{\rho u_\infty s}{\mu}$$

$s \rightarrow$ distance from leading edge

u_∞ - Bulk fluid velocity

Critical Reynolds number = 3×10^5

* Transition length for laminar & turbulent flow



fully developed flow

$\lambda_t \rightarrow$ Transition length.

* Transition length:- length necessary to achieve fully developed flow.

* If the flow is laminar :-

$$\frac{\lambda_t}{\Delta} = 0.05 N_{Re}$$

* for Turbulent flow

$$\lambda_t = (40-50) \Delta$$

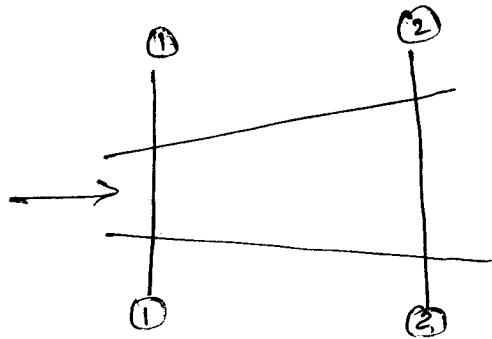
Independent of (Re)

* Rate of flow OR Discharge (Q)

$$[Q = u \times A],$$

* Continuity equation :-

The equation based on the principle of conservation of mass is called continuity equation. Thus for a fluid flowing through the pipe at all the cross-section, the quantity of fluid per second is constant.



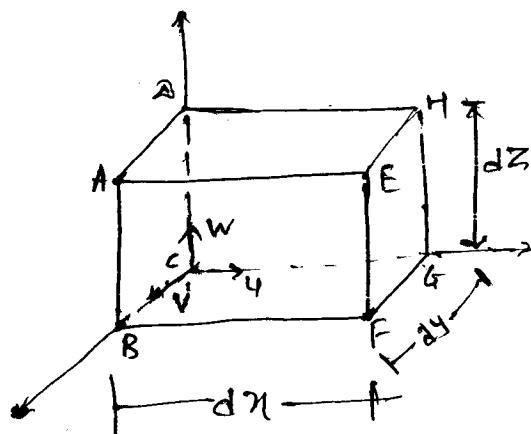
∴ According to law of conservation of mass

$$\rho_1 A_1 u_1 = \rho_2 A_2 u_2$$

for incompressible fluid

$$[A_1 u_1 = A_2 u_2],$$

* Continuity equation in three dimensions



* Mass of fluid entering the face ABCD per second.

$$= \rho \times \text{velocity in } (\eta \text{ direction}) \times \text{Area of ABCD}$$

$$= \rho \times u \times (dy \times dz)$$

Then mass of the fluid leaving the face EFGH per sec

$$= \rho u dy dz + \frac{\delta}{\delta \eta} (\rho u dy dz) d\eta$$

\therefore Gain of mass in η -direction

$$= \text{Mass through ABCD} - \text{Mass through EFGH per sec.}$$

$$= \rho u dy dz - \rho u dy dz - \frac{\delta}{\delta \eta} (\rho u dy dz) d\eta$$

$$= - \frac{\delta}{\delta \eta} (\rho u dy dz) d\eta$$

Similarly for y -direction :-

$$= - \frac{\delta}{\delta y} (\rho v d\eta dz) dy$$

for z -direction :-

$$= - \frac{\delta}{\delta z} (\rho w d\eta dy dz)$$

* Net gain of masses :-

$$= - \left[\frac{\delta}{\delta \eta} (\rho u) + \frac{\delta}{\delta y} (\rho v) + \frac{\delta}{\delta z} (\rho w) \right] d\eta dy dz$$

Since the mass is neither created nor destroyed in the fluid element, the net increase of mass per unit time in the fluid element must be equal to the rate of increase of mass of fluid in the element. But mass of fluid in the element is $\rho dV dy dz$ and its rate of increase with time is $\frac{\delta}{\delta t} (\rho dV dy dz)$ or $\frac{\delta \rho}{\delta t} (dV dy dz)$

\therefore Equating two expressions

$$-\left[\frac{\delta}{\delta x} (\rho u) + \frac{\delta}{\delta y} (\rho v) + \frac{\delta}{\delta z} (\rho w) \right] dV dy dz = \frac{\delta \rho}{\delta t} dV dy dz$$

$$\boxed{\frac{\delta \rho}{\delta t} + \frac{\delta}{\delta x} (\rho u) + \frac{\delta}{\delta y} (\rho v) + \frac{\delta}{\delta z} (\rho w) = 0}$$

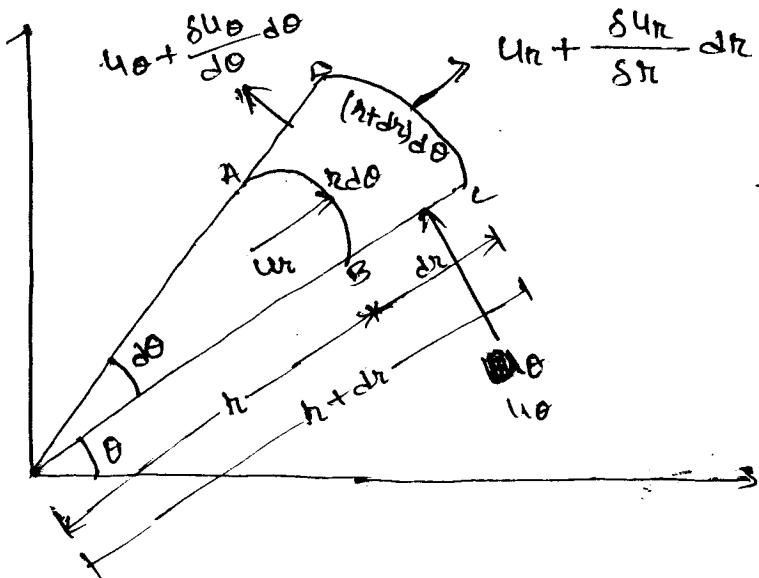
This is the continuity equation in cartesian co-ordinates in its most general form. This eqn is applicable to

- (1) Steady state & unsteady flow
- (2) Uniform & non-uniform flow
- (3) Compressible & incompressible flow (fluids)

for steady & incompressible flow

$$\boxed{\frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} + \frac{\delta w}{\delta z} = 0}$$

* Continuity equation in Cylindrical polar co-ordinates.



* Velocity & Acceleration:-

$$V = u\hat{i} + v\hat{j} + w\hat{k} = \sqrt{u^2 + v^2 + w^2}$$

$$\therefore a_n = \frac{du}{dt} = \frac{\delta u}{\delta r} \frac{dr}{dt} + \frac{\delta u}{\delta \theta} \frac{d\theta}{dt} + \frac{\delta u}{\delta z} \frac{dz}{dt} + \frac{\delta u}{\delta t}$$

$$\text{But } \frac{dr}{dt} = u, \quad \frac{d\theta}{dt} = v, \quad \frac{dz}{dt} = w$$

$$\therefore a_n = \frac{du}{dt} = u \frac{\delta u}{\delta r} + v \frac{\delta u}{\delta \theta} + w \frac{\delta u}{\delta z} + \frac{\delta u}{\delta t}$$

$$a_y = \frac{dv}{dt} = u \frac{\delta v}{\delta r} + v \frac{\delta v}{\delta \theta} + w \frac{\delta v}{\delta z} + \frac{\delta v}{\delta t}$$

$$a_z = \frac{dw}{dt} = u \frac{\delta w}{\delta r} + v \frac{\delta w}{\delta \theta} + w \frac{\delta w}{\delta z} + \frac{\delta w}{\delta t}$$

Steady state

$$\boxed{\frac{\delta v}{\delta t} = 0}$$

$$\therefore A = a_n \hat{i} + a_y \hat{j} + a_z \hat{k}$$

$$= \sqrt{a_n^2 + a_y^2 + a_z^2}$$

* Velocity potential function & stream function

① Velocity potential function:-

$$u = -\frac{\partial \phi}{\partial x}, \quad v = \frac{\partial \phi}{\partial y}, \quad w = -\frac{\partial \phi}{\partial z}$$

* Velocity components in cylindrical polar coordinates

$$u_r = \frac{\partial \phi}{\partial r}$$

$$u_\theta = \frac{1}{r} \frac{\partial \phi}{\partial \theta}$$

u_r = Velocity component in radial direction

u_θ = Velocity component in tangential direction

The continuity equation for an incompressible steady flow is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Substitute the value of $u, v \& w$

$$\frac{\partial}{\partial x} \left(-\frac{\partial \phi}{\partial x} \right) + \frac{\partial}{\partial y} \left(-\frac{\partial \phi}{\partial y} \right) + \frac{\partial}{\partial z} \left(-\frac{\partial \phi}{\partial z} \right) = 0$$

$$\therefore \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$



Laplace equation

* Properties of the Potential function:-

The rotational components are given by

$$w_z = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$

$$w_y = \frac{1}{2} \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right)$$

$$w_x = \frac{1}{2} \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right)$$

$$w_z = w_y = w_x = 0$$

When rotational components are zero, the flow is called irrotational. Hence the properties of the potential function are.

1. If velocity potential (ϕ) exists, the flow should be irrotational
2. If velocity potential (ϕ) satisfy the Laplace equation, it represents the possible steady incompressible irrotational flow.

* Stream function :-

$$\Psi = f(x, y)$$

$$\frac{\delta \Psi}{\delta x} = v \quad \& \quad \frac{\delta \Psi}{\delta y} = -u$$

The velocity components in cylindrical polar co-ordinates in terms of stream function are given as .

$$u_r = \frac{1}{r} \frac{\delta \Psi}{\delta \theta} \quad \& \quad u_\theta = -\frac{\delta \Psi}{\delta r}$$

. ∵ Continuity eqn in two dimensional flow is

$$\frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} = 0$$

$$\frac{\delta}{\delta x} \left(\frac{-\delta \Psi}{\delta y} \right) + \frac{\delta}{\delta y} \left(\frac{\delta \Psi}{\delta x} \right) = 0$$

$$\text{or } \rightarrow \frac{\delta^2 \Psi}{\delta x \delta y} + \frac{\delta^2 \Psi}{\delta y \delta x} = 0$$

$$w_z = \frac{1}{2} \left(\frac{\delta v}{\delta x} - \frac{\delta u}{\delta y} \right) = \frac{1}{2} \left(\frac{\delta^2 \Psi}{\delta x^2} + \frac{\delta^2 \Psi}{\delta y^2} \right)$$

for irrotational flow

$$w_z = 0 \quad \therefore \quad \boxed{\frac{\delta^2 \Psi}{\delta x^2} + \frac{\delta^2 \Psi}{\delta y^2} = 0}$$

* Relation b/w stream function & velocity potential

$$u = -\frac{\delta \phi}{\delta n} \quad \& \quad v = -\frac{\delta \phi}{\delta y}$$

$$\& \quad u = -\frac{\delta \psi}{\delta y} \quad \& \quad v = \frac{\delta \psi}{\delta n}$$

$$\therefore \boxed{\frac{\delta \phi}{\delta n} = \frac{\delta \psi}{\delta y} \quad \& \quad \frac{\delta \phi}{\delta y} = -\frac{\delta \psi}{\delta n}}$$

Unit - III

① Equation of motion:

*

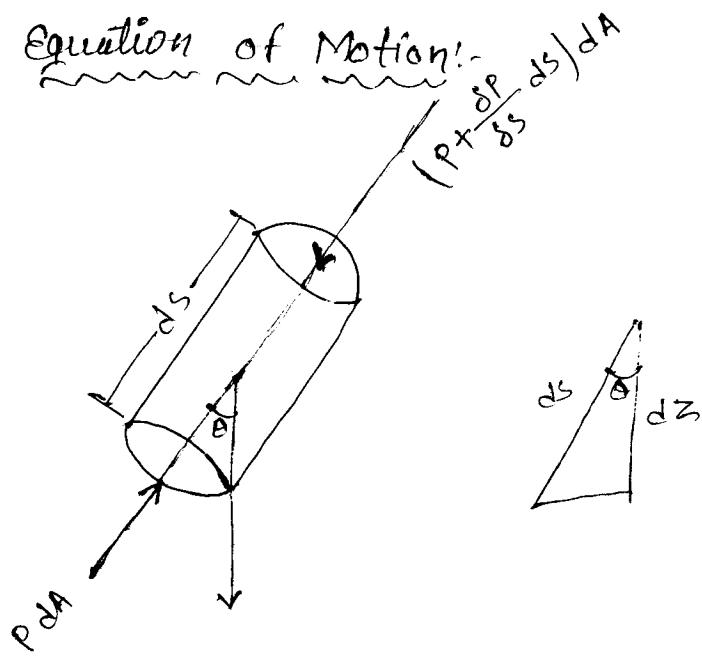
Net force

$$F_n = m a_n$$

$$\therefore F_n = \vec{F_g} + \vec{F_p} + \vec{F_v} + \vec{F_t} + \vec{F_c}$$

- (i) If F_c is negligible eqn is called Reynold's eqn of motion.
- (ii) If F_t is negligible eqn is called Navier-Stokes equation.
- (iii) If the flow is assumed to be ideal viscous force is zero.
⇒ eqn of motions are known as Euler's eqn of motion.

* Euler's Equation of Motion:



* This is the eqn of motion in which the forces due to gravity & pressure are taken into consideration

Cylindrical element

$$\text{Cross section} = dA$$

$$\text{length} = ds$$

Force acting

(i) Pressure force in the direction of flow. = PdA

2. Pressure force in the opposite direction = $(P + \frac{dp}{ds} ds)dA$

3. weight of element = $\rho g dA ds$

$$\rho dA ds \cdot a_s = P dA - \left(P + \frac{dp}{ds} ds \right) dA - \rho g dA ds \cos\theta$$

$$\rho dA ds \times a_s = P dA - P dA - \frac{dp}{ds} ds dA - \rho g dA ds \cos\theta$$

$$\therefore a_s = \frac{dv}{dt} \quad \text{where } v \text{ is a function of } s \text{ & } t$$

$$\therefore a_s = \frac{\delta v}{\delta s} \frac{ds}{dt} + \frac{\delta v}{\delta t} = v \frac{\delta v}{\delta s} + \frac{\delta v}{\delta t}$$

$$\text{if the flow is steady } \frac{\delta v}{\delta t} = 0$$

$$a_s = v \frac{\delta v}{\delta s}$$

$$\therefore -\frac{\delta p}{\delta s} ds dA - \rho g dA ds \cos\theta = \rho dA ds \times \frac{\delta v}{\delta s}$$

dividing by $\rho dA ds$

$$\therefore \frac{\delta p}{\delta s} + g \cos\theta + v \frac{\delta v}{\delta s} = 0$$

$$\cos\theta = \frac{dz}{ds}$$

$$\frac{1}{g} \frac{dp}{ds} + g \frac{dz}{ds} + v \frac{dv}{ds} = 0$$

$$\boxed{\frac{dp}{g} + g dz + v dv = 0}$$

* Bernoulli's equation

It is obtained by integrating the Euler's eqn of motion.

$$\int \frac{dp}{g} + \int g dz + \int v dv = \text{const.}$$

If flow is incompressible ρ is constant

$$\frac{P}{g} + gz + \frac{v^2}{2} = \text{const.}$$

$$\boxed{\frac{P}{\rho g} + z + \frac{v^2}{2g} = \text{const}}$$

* Assumption for Bernoulli's eqn

- (i) The fluid is ideal ($\mu=0$)
- (ii) The flow is steady
- (iii) The flow is incompressible
- (iv) The flow is irrotational

* Bernoulli's eqn for real fluid

$$\left[\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_L \right]$$

h_L = loss of energy b/w 1 & 2

* Mass velocity:-

$$G = \bar{V} \rho = \frac{m}{S}$$

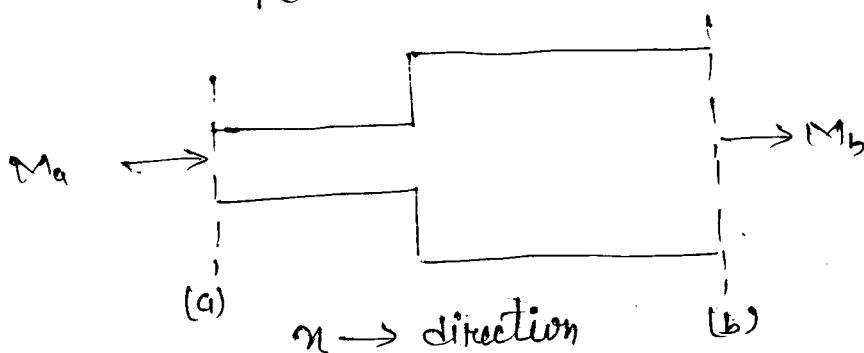
$$G \rightarrow \text{kg/sm}^2$$

* The advantage of using G is that it is independent of temp. & pressure when the flow is steady & cross section is unchanged. especially useful when compressible fluids are considered.

* Macoscopic Momentum balance

$$\sum F = (M_b - M_a)$$

Momentum balance



* Momentum of total stream. Momentum correction factor

- ⇒ Momentum flow rate → M
- ⇒ Mass flow rate → m
- ⇒ Velocity → u

$$M = mu$$

If u varies from point to point in the cross section of the stream, however, the total momentum flow does not equal the product of the mass flow rate & the avg velocity or $m\bar{v}$. in general it is somewhat greater than this.

The necessary correction factor is best found from the convective momentum flux i.e the momentum carried by the moving fluid through a unit cross-sectional area of the channel in a unit time. This is the product of the linear velocity normal to the cross-section and the mass velocity (or mass flux) for a differential cross-sectional area ds , the momentum flux is

$$\frac{dM}{ds} = (\beta u)u = \beta u^2$$

$$\left. \begin{aligned} m &= uss \\ \frac{m}{s} &= us \\ \text{flux} &= us \end{aligned} \right\}$$

The momentum flux of the whole stream, for a constant density fluid

$$\frac{M}{s} = \frac{\beta \int u^2 ds}{s}$$

$M \rightarrow$ Momentum flow rate

The momentum correction factor β is defined as

$$\boxed{\beta = \frac{M/s}{s \bar{v}^2}}$$

$$\left. \begin{aligned} \beta &= 4/3 \text{ for laminar} \\ \beta &= 1 \text{ for Turbulent} \end{aligned} \right\}$$

$$\boxed{\beta = \frac{1}{2} \int \left(\frac{u}{\bar{v}} \right)^2 ds}$$

Time

* To find β for any given flow situation, the variation of u with position in the cross section must be known.

* Kinetic energy of stream

$$dK_E = (su ds) \frac{u^2}{2}$$

$$dK_E = \frac{s u^3 ds}{2}$$

where K_E represents the time rate of flow of kinetic energy.

The total rate of flow of kinetic energy through the entire cross section s is, assuming cons. density within the area s

$$K_E = \frac{s}{2} \int u^3 ds$$

\therefore Kinetic energy per unit mass of flowing fluid

$$\frac{K_E}{m} = \frac{\frac{s}{2} \int u^3 ds}{s \sqrt{s}} = \frac{\frac{1}{2} \int u^3 ds}{\sqrt{s}}$$

* Kinetic energy correction factor :-

$$\frac{\alpha \bar{V}^2}{2} = \frac{E_K}{m} = \frac{\int u^3 ds}{2 \cdot \sqrt{s}}$$

$$\alpha = \frac{\int u^3 ds}{\bar{V}^3 s}$$

if α is known use avg. velocity instead of u .

$\alpha = 2$ for laminar

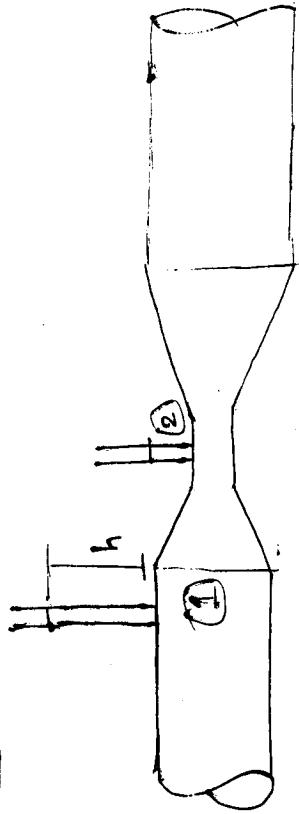
$\alpha = 1.05$ Turbulent

3rd किंतु 2nd

Equation:

* Practical Application of Bernoulli's equation:-

1 Venturi meter:-



d_1 = Diameter of inlet section

P_1 = Pressure at section (1)

V_1 = Velocity of fluid at section (1)

$$\therefore Q = \frac{\pi}{4} d_1^2 V_1$$

Same d_2 , P_2 , V_2 are corresponding values at (2)

Applying Bernoulli's eqn at (1) & (2)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

for horizontal pipe ($Z_1 = Z_2$)

$$\frac{P_1 - P_2}{\rho g} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$$\frac{P_1 - P_2}{\rho g} = h$$

$$\therefore h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

applying continuity eqn

$$Q_1 V_1 = Q_2 V_2$$

$$\therefore V_1 = \frac{Q_2 V_2}{Q_1}$$

$$h = \frac{V_2^2}{2g} - \frac{(C_{12} V_2 / a_1)^2}{2g}$$

$$h = \frac{V_2^2}{2g} \left[1 - \frac{a_2^2}{a_1^2} \right] = \frac{V_2^2}{2g} \left[\frac{a_1^2 - a_2^2}{a_1^2} \right]$$

$$V_2^2 = 2gh \cdot \frac{a_1^2}{a_1^2 - a_2^2}$$

$$V_2 = \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

$$\boxed{Q = a_2 V_2 = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}}$$

Ideal condition

Theoretical discharge

Actual discharge will be less than theoretical discharge.

$$\boxed{Q_{act} = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}}$$

$C_d \Rightarrow$ Coefficient of venturimeter & its value is less than 1

* Value of 'h' given by differential U-tube manometer

Case I: Let the differential manometer contains a liquid which is heavier than the liquid flowing through the pipe.

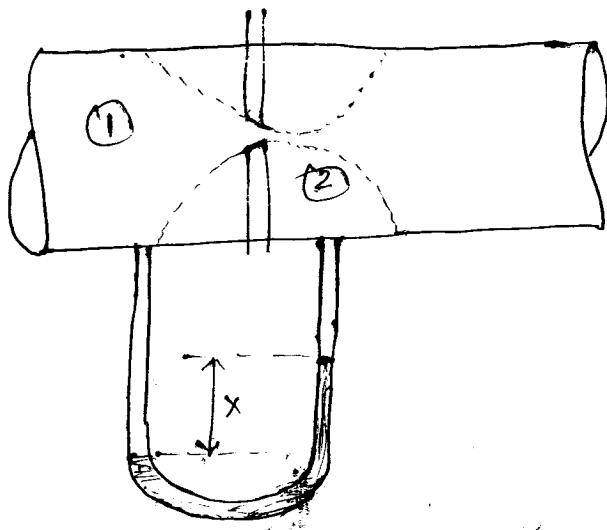
$S_h \rightarrow$ Sp. gr. of the heavier liquid

S_o - Sp. gr. of liquid (pipe)

$n \rightarrow$ Difference of the heavier liquid column in U-tube

$$\boxed{h = n \left[\frac{S_h}{S_o} - 1 \right]}$$

* Orifice Meter :-



Applying Bernoulli's eqn

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\left(\frac{P_1}{\rho g} + z_1 \right) - \left(\frac{P_2}{\rho g} + z_2 \right) = h$$

$$\therefore h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$$V_2 = \sqrt{2gh + V_1^2}$$

a_2 → Area of vena contracta

a_o → Area of orifice

c_c → Coefficient of contraction

$$\therefore C_c = \frac{a_2}{a_o}$$

$$\therefore a_2 = C_c \times a_o$$

* Continuity eqn

$$a_1 V_1 = a_2 V_2$$

$$a_1 V_1 = a_o C_c V_2$$

$$V_1 = \frac{a_o C_c V_2}{a_1}$$

$$\therefore V_2 = \sqrt{2gh + \frac{a_0^2 c_c^2 V_2^2}{a_1^2}}$$

$$V_2^2 = 2gh + \frac{a_0^2 c_c^2 V_2^2}{a_1^2}$$

$$2gh = V_2^2 \left[1 - \left(\frac{a_0}{a_1} \right)^2 c_c^2 \right]$$

$$V_2 = \frac{\sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1} \right)^2 c_c^2}}$$

$$Q = V_2 a_2 = V_2 \times a_0 c_c$$

$$Q = \frac{a_0 c_c \sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1} \right)^2 c_c^2}}$$

The above expression is simplified by using

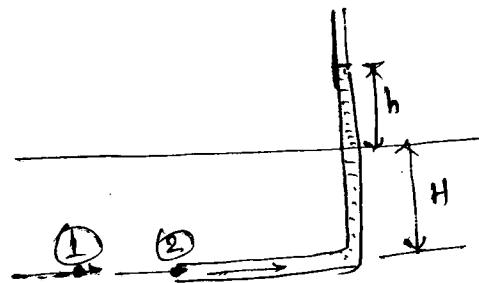
$$C_d = C_c \frac{\sqrt{1 - \left(\frac{a_0}{a_1} \right)^2}}{\sqrt{1 - \left(\frac{a_0}{a_1} \right)^2 c_c^2}}$$

$$C_c = C_d \frac{\sqrt{1 - \left(\frac{a_0}{a_1} \right)^2 c_c^2}}{\sqrt{1 - \left(\frac{a_0}{a_1} \right)^2}}$$

$$Q = a_0 C_d \frac{\sqrt{1 - \left(\frac{a_0}{a_1} \right)^2 c_c^2}}{\sqrt{1 - \left(\frac{a_0}{a_1} \right)^2}} \times \frac{\sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1} \right)^2 c_c^2}}$$

$$\Rightarrow Q = \frac{C_d a_0 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}$$

* Pitot tube:-



$H \rightarrow$ Depth of tube in the liquid

$h \rightarrow$ Rise of liquid in the tube above the free surface

Applying Bernoulli's eqn

$$\frac{P_1}{\rho g} + z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + z_2 + \frac{V_2^2}{2g}$$

$$[z_1 = z_2],$$

Point 2 is stagnation point $\therefore [V_2 = 0]$ Imp

$\frac{P_1}{\rho g}$ = Pressure head at (1) = H

$\frac{P_2}{\rho g}$ = Pressure head at (2) = $(H+h)$

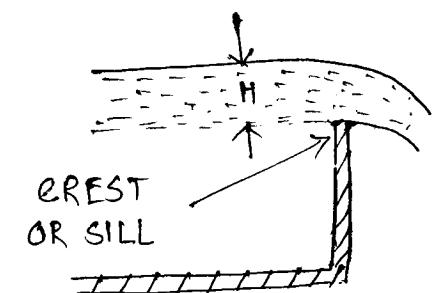
$$H + \frac{V_1^2}{2g} = h + H$$

$$[V_1 = \sqrt{2gh}], \text{ Theoretical velocity}$$

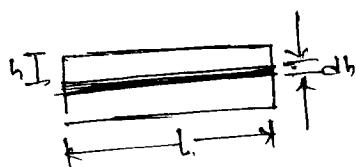
$$[V_{ac} = C_v \sqrt{2gh}], \text{ Actual velocity}$$

* Notches & weirs:-

* Discharge over a rectangular Notch or weir.



(a) Rectangular notch



(c) Section at crest

H = Head of water over the crest.

L = length of Notch or weir

Consider elementary horizontal strip of water of thickness dh & length L at a depth h from the free surface of water. ~~at~~

$$\Rightarrow \text{Area of strip} = L \times dh$$

$$\text{Theoretical velocity} = \sqrt{2gh}$$

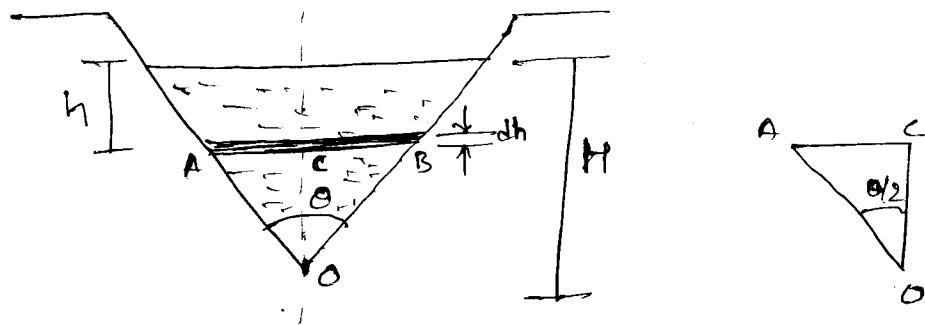
$$\begin{aligned}\text{Discharge } Q &= C_d \times \text{Area} \times \text{velocity} \\ &= C_d \times L \times dh \times \sqrt{2gh}\end{aligned}$$

$$\therefore Q = \int_0^H C_d L dh \sqrt{2gh} = C_d L \sqrt{2g} \int_0^H h^{1/2} dh$$

$$Q = C_d \times L \times \sqrt{2g} \left[\frac{h^{3/2}}{3/2} \right]_0^H$$

$$Q = \frac{2}{3} C_d L \sqrt{2g} H^{3/2}$$

* Discharge over a triangular notch or weir



H = head of water above the V-notch

θ = angle of notch

Consider horizontal strip of water of thickness dh at a depth of h from the free surface of water

$$\tan \frac{\theta}{2} = \frac{AC}{OC} = \frac{Ac}{(H-h)}$$

$$AC = (H-h) \tan \theta/2$$

$$AB = 2AC = 2(H-h) \tan \theta/2$$

$$\therefore \text{Area of strip} = (2(H-h) \tan \theta/2 \times dh)$$

$$\text{Theoretical velocity} = \sqrt{2gh}$$

$$dQ = Cd \times 2(H-h) \tan \theta/2 \times dh \times \sqrt{2gh}$$

$$Q = \int_0^H Cd 2(H-h) \tan \theta/2 \sqrt{2gh} dh$$

$$Q = 2 Cd \times \tan \theta/2 \times \sqrt{2g} \int_0^H (H-h)^{1/2} h^{3/2} dh$$

$$Q = 2 Cd \times \tan \theta/2 \sqrt{2g} \int_0^H (Hh^{3/2} - h^{5/2}) dh$$

$$Q = 2 Cd \times \tan \theta/2 \sqrt{2g} \left[\frac{Hh^{5/2}}{5/2} - \frac{h^{5/2}}{5/2} \right]_0^H$$

$$Q = 2 \times C_d \times \tan \frac{\theta}{2} \times \sqrt{2g} \left[\frac{2}{3} H^{5/2} - \frac{2}{5} H^{5/2} \right]$$

$$Q = 2 \times C_d \times \tan \frac{\theta}{2} \times \sqrt{2g} \left[\frac{4}{15} H^{5/2} \right]$$

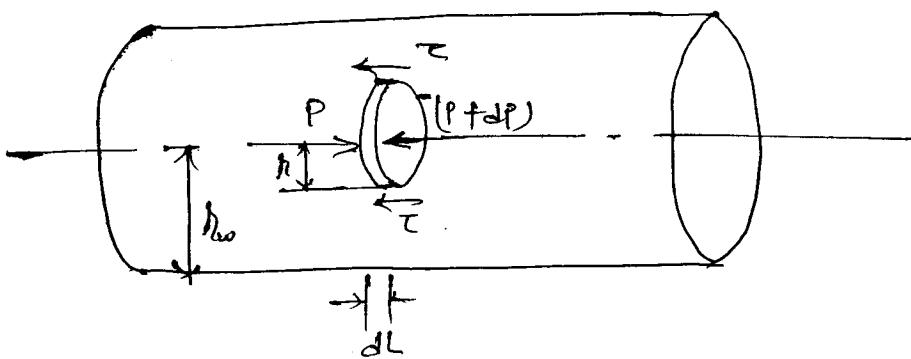
$$Q = \frac{8}{15} C_d \tan \frac{\theta}{2} \sqrt{2g} \times H^{5/2}$$

Unit-5

* Flow of Incompressible fluid in pipes :-

* Shear stress distribution in a cylindrical tube :-

- (i) steady flow (iii) fully developed.
- (ii) density cons.



Since the flow is fully developed • ($B_a = B_b$)

& $\bar{V}_b = \bar{V}_a$ • so that $\sum F = 0$ •

$$S_a = S_b = \pi r^2$$

$$\sum F = \pi r^2 P - \pi r^2 (P + dP) - \sum \pi r dL \tau = 0$$

Simplifying this eqn & dividing by $\pi r^2 dL$ gives

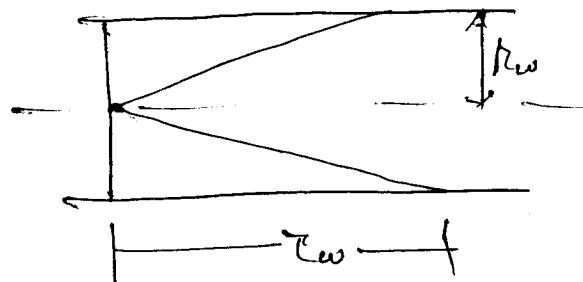
$$\frac{dP}{dL} + \frac{\sum \tau}{r} = 0 \quad \rightarrow \textcircled{1}$$

* In steady flow, either laminar or turbulent, the pressure at any given cross section of a stream tube is constant, so that $\frac{dP}{dL}$ is independent of r . & (1) can be written for the entire cross-section $\tau = \tau_w$ & $r = r_w$

$$\frac{dP}{dL} + \frac{\sum \tau_w}{r_w} = 0 \quad \rightarrow \textcircled{2}$$

$$\left[\frac{\tau_w}{r_w} = \frac{\tau}{r} \right]_{1/1}$$

Sub. (2-1)



Shear stress τ

* Variation of shear stress in pipe

* Relation b/w skin friction & wall shear:-

$$\Delta P = P_a - P_b$$

$$P_a = P, \quad P_b = P - \Delta P$$

\therefore from bernoulli eqn

$$\frac{P}{\rho g} = \frac{P - \Delta P}{\rho g} + h_{fs}$$

$$\frac{\Delta P}{\rho g} = h_{fs}$$

* for definite length of pipe $\frac{\Delta P}{\Delta L}$ becomes $\frac{\Delta P}{\Delta L}$.

$$h_{fs} = \frac{4 T_w}{\rho D} \frac{\Delta P}{\Delta L} \quad \left. \right\} D = \text{diameter of pipe}$$

* The friction factor:-

⇒ Useful in the study of turbulent flow.

* Defined as the ratio of the wall shear stress to the product of the density & the velocity head $\cdot \frac{V^2}{2}$

$$\therefore f = \frac{\tau_w}{\rho \frac{V^2}{2}} = \frac{2 \tau_w}{\rho V^2}$$

$$h_{fs} = \frac{4 \tau_w \Delta L}{\rho g} \quad \checkmark$$

$$\tau_w = \frac{\rho V^2 f}{2}$$

$$h_{fs} = \frac{4 f \Delta L \bar{V}^2}{2 \rho g}$$

(1) form friction $\rightarrow h_f$
 (2) skin friction $\rightarrow h_{fs}$

for Boundary layer separation $h_f \gg h_{fs}$

* Laminar flow in pipes :-

The local velocity u depends upon the radius r .

$$\therefore dS = 2\pi r dr$$

dS - Area of elementary ring } width dr
 r - Radius }

$$\therefore \mu = - \frac{T}{du/dr} \quad \left. \right\} \text{(-ve sign indicates } u \downarrow r \uparrow)$$

$$\therefore \frac{du}{dr} = - \frac{T}{\mu}$$

from eq

$$\frac{du}{dr} = -\frac{\tau_w r}{\mu_w \mu}$$

Boundary condition $u=0$ at $r=r_w$

$$\int_0^r du = -\frac{\tau_w}{\mu_w \mu} \int_{r_w}^r r dr$$

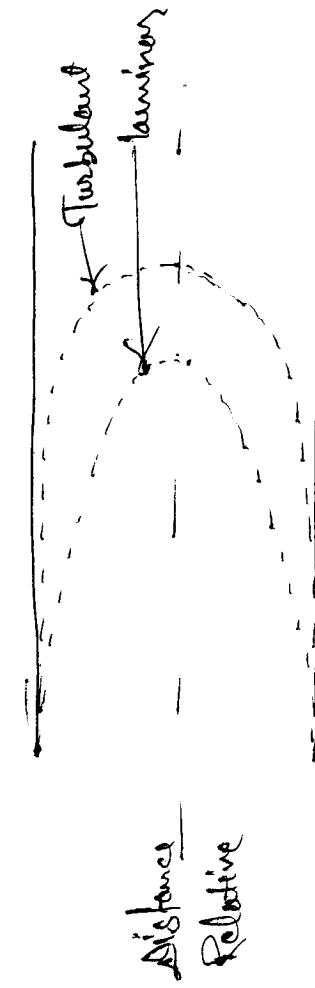
$$u = \left[\frac{\tau_w}{2 \mu_w \mu} \left(r_w^2 - r^2 \right) \right]^{1/2}$$

$$r=0 \quad u=u_{max}$$

$$u_{max} = \frac{\tau_w r_w}{2 \mu}$$

$$\frac{u}{u_{max}} = 1 - \left[\frac{r}{r_w} \right]^2$$

The velocity distribution with respect to radius is a parabola with the apex at the centerline of the pipe.



Fraction of max. velocity

* Average velocity :-

$$\bar{V} = \frac{m}{\rho s} = \frac{1}{s} \int u \, ds$$

$\left. \right\} s - \text{cross sectional area}$

$$\therefore \bar{V} = \frac{\tau_{tw}}{r_w^3 \mu} \int_0^{r_w} (r_w^2 - r^2) / \pi \, dr$$

$$\boxed{\bar{V} = \frac{\tau_{tw} r_w}{4 \mu}}$$

$$\boxed{\frac{\bar{V}}{V_{max}} = 0.5}$$

* Hagen - Poiseuille Equation

$$\bar{V} = \frac{\tau_{tw} r_w}{4 \mu}$$

$$\left. \right\} h_{fs} = \frac{4 \tau_{tw} \Delta L}{3 g A}$$

$$\cancel{\Phi} = \frac{2 \tau_{tw}}{3 r_w} \Delta L = \frac{\Delta P_s}{S}$$

$$\bar{V} = \frac{\Delta P_s D^2}{32 \Delta L \mu}$$

$$\boxed{\Delta P_s = \frac{32 \mu \bar{V} \Delta L}{D^2}}$$

$$\& \text{Science} \quad \Delta P_s = \frac{4 \tau_{tw}}{D \cancel{\Phi}} \Delta L$$

$$\left. \right\} f = \frac{2 \tau_{tw}}{S \bar{V}^2}$$

$$\frac{32 \mu \bar{V} \Delta L}{D^2} = \frac{4 \tau_{tw}}{\cancel{D \Phi}}$$

$$\frac{4 \tau \Delta L}{D} = \frac{32 \mu \bar{V} \Delta L}{D^2}$$

$$\boxed{\tau_w = \frac{8 \sqrt{\mu}}{A}}$$

$$\boxed{f = \frac{2 \tau_w}{g v^2} = \frac{16 M}{A \bar{v} s} = \frac{16}{Re}} //$$

* Velocity distribution for turbulent flow

It is customary to express the velocity distribution in turbulent flow not as velocity vs. distance but in terms of dimensionless parameters defined by the following eqn

$$u^* = \sqrt{\frac{F}{2}} = \sqrt{\frac{\tau_w}{\rho}}$$

$$u^+ = \frac{u}{u^*} \quad \& \quad y^+ = \frac{y u^* \rho}{\mu} = \frac{y}{\mu} \sqrt{\tau_w \rho}$$

u^* = friction velocity, u^+ = velocity ratio

y^+ = distance (dimensionless)

y = Distance from wall of tube

* Relationship b/w y , r & τ_w is

$$\boxed{r_w = r + y},,$$

Note that ~~r~~ y^+ may be considered to be a Reynolds number based on friction velocity & distance from wall.

* Universal velocity distribution equation :-

Since the viscous sublayer is very thin, $\tau = \tau_w$, & so

$$\frac{du}{dy} = \frac{\tau_w}{\mu}$$

$$u = u^+ u^*$$

$$y = \frac{u^+ y^3}{\mu}$$

$$du = u^+ du^* + u^* du^*$$

$$\frac{dy^+}{dy} = \frac{u^* s}{\mu}$$

$$\frac{\mu dy^+}{u^* s} = dy$$

$$\mu s \frac{u^* du^+}{\mu dy^+} = \frac{\tau_w}{\mu}$$

$$\frac{du^+}{dy^+} = \frac{\tau_w}{s u^{*2}}$$

* friction factor in flow through channels of Non circular cross-sections.

$$d_{eq} = \sqrt{\frac{\text{Cross sectional area}}{\text{wetted perimeter}}}$$

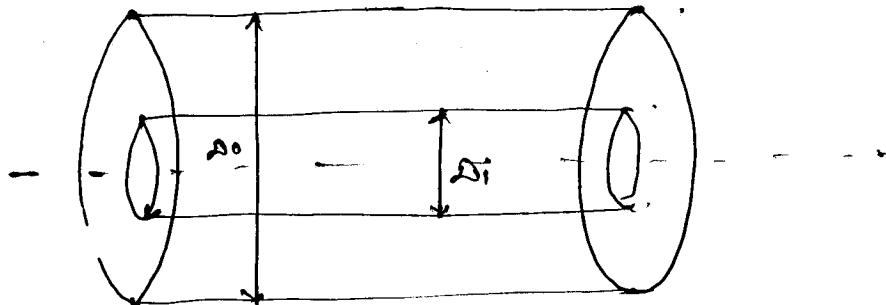
for Circular tube

$$d_{eq} = \frac{4\pi/4\alpha^2}{\pi\alpha} = \alpha$$

* for annulus:

$$d_{eq} = \frac{\frac{\pi D_o^2}{4} - \frac{\pi d_i^2}{4}}{\pi d_i + \pi D_o}$$

D_o & d_i inside & outside diameter of annulus

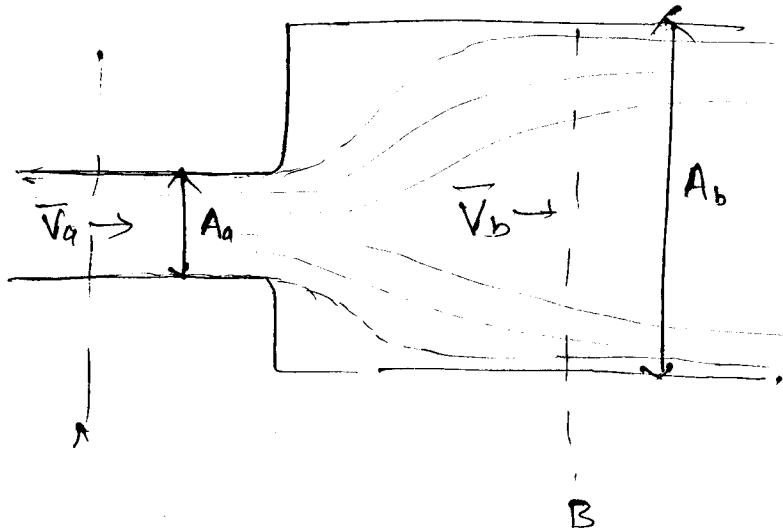


* friction losses due to sudden expansion of cross-section:

The friction losses h_{fe} from a sudden expansion of cross section is proportional to the velocity v the smaller head of the fluid in the small conduit.

$$h_{fe} = k_e \frac{v^2}{2g}$$

where $k_{fe} \rightarrow$ Expansion loss coefficient
 $\bar{V} \rightarrow$ Avg velocity in smaller, or upstream conduit.



Apply Bernoulli eqn

$$\frac{P_a - P_b}{\rho g} = \frac{V_b^2 - V_a^2}{2g} + h_{fe}$$

$$h_{fe} =$$

Rate of change of momentum = force

$$(P_a A_a - P_b A_b) = m \bar{V}_b - m \bar{V}_a$$

$$\left(-\frac{P_a}{\rho V_a} - \frac{P_b}{\rho V_b} \right) = \bar{V}_b - \bar{V}_a$$

∴ force apply

$$F_n = P_a A_a + P' (A_b - A_a) \doteq P_b A_b$$

But experimentally it is found that $P' = P_a$

$$F_n = P_a A_a + P_a A_b - P_a A_a = P_b A_b$$

$$F_n = P_a A_b - P_b A_b = (P_a - P_b) A_b$$

\therefore Rate of change of momentum = mV

$$\therefore = \rho A_a V_a \times V_a = \rho A_a V_a^2 \quad \text{--- at (a)}$$

$$\text{so at b} = \rho A_b V_b^2$$

$$\therefore \text{Change in momentum} = \rho (A_b V_b^2 - A_a V_a^2)$$

Apply continuity eqn

$$A_a V_a = A_b V_b$$

\therefore Change in momentum

$$= \rho A_b V_b^2 - \rho \times \frac{A_b V_b}{V_a} \times V_a^2.$$

$$= \rho A_b V_b^2 - \rho A_b V_a V_b$$

$$= \rho A_b (V_b^2 - V_a V_b)$$

$$\therefore (P_a - P_b) A_b = \rho A_b (V_b^2 - V_a V_b)$$

$$\frac{P_a - P_b}{\rho} = V_b^2 - V_a V_b$$

$$\therefore \frac{P_a}{\rho g} - \frac{P_b}{\rho g} = \frac{V_b^2 - V_a V_b}{g}$$

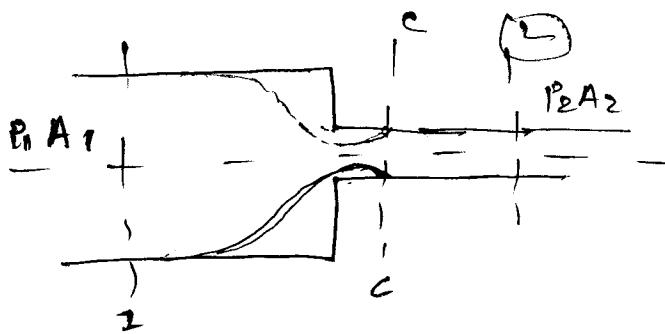
$$\therefore h_c = \frac{V_b^2 - V_a V_b}{g} + \frac{V_a^2}{2g} - \frac{V_b^2}{2g}$$

$$h_{cf} = \left[\frac{(V_a - V_b)^2}{2g} \right]$$

$$\therefore h_{fe} = \frac{V_a^2}{2g} \left(1 - \frac{A_a}{A_b}\right)^2$$

$$\therefore K_e = \left(1 - \frac{A_a}{A_b}\right)^2$$

* Sudden contraction ..



$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_c$$

* A_c = Area of flow at section C-C

V_c = Velocity of flow at section C-C

A_2 = Area of flow at section Z-Z

h_c = Loss of head due to sudden contraction.

Now h_c = Actual loss of head due to enlargement from section C-C to section Z-Z and is given by

eqn

$$h_c = \frac{(V_c - V_2)^2}{2g} = \frac{V_2^2}{2g} \left[\frac{V_c}{V_2} - 1 \right]^2$$

$$A_c V_c = A_2 V_2$$

$$\frac{V_c}{V_2} = \frac{A_2}{A_c}$$

$$h_c = \frac{V_2^2}{2g} \left[\frac{A_2}{A_c} - 1 \right]^2$$

} If A_c is not given
head loss due to contraction
is taken as

$$h_c = 0.5 \frac{V_2^2}{2g}$$

* The Effect of fittings and valves :-

$$h_{ff} = k_f \frac{V_a^2}{2g}$$

V_a = avg velocity in pipe leading to fitting

K value for valves & fittings

$$K = f \frac{L}{D}$$

UNIT-5

* What is Pump?

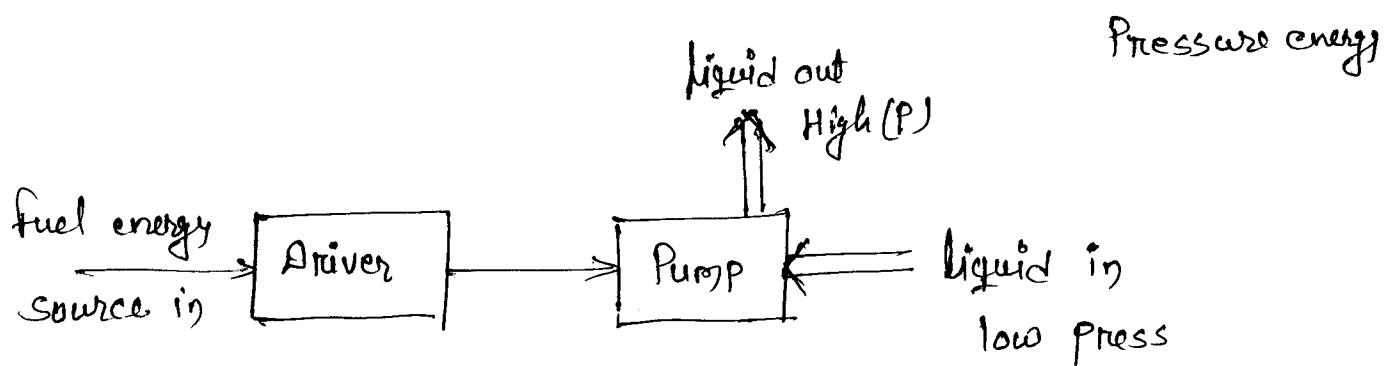
A Pump is a machine used to move liquid through a piping system & to raise the pressure of the liquid.

A pump can be further defined as a machine that uses several energy transformations to increase the pressure of a liquid.

Input energy \rightarrow rotating mechanical energy \rightarrow kinetic energy



Pressure energy



* Why increase a liq. pressure?

(1) Static elevation

(2) friction \rightarrow To overcome frictional losses

(3) Pressure \rightarrow Necessary to overcome a vacuum in the supply vessel.

④ Velocity :- There is another factor to be considered here namely that not all of the velocity energy in a pump is converted to potential or pressure energy.

outlet or discharge dia \ll suction dia

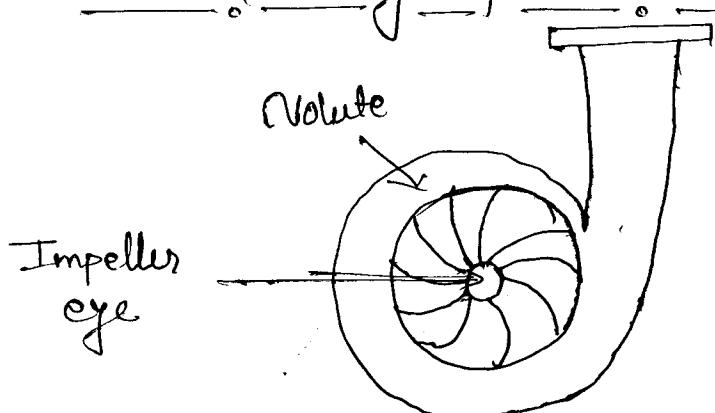
* Classification of pumps :-

(i) Kinetic (ii) Positive displacement

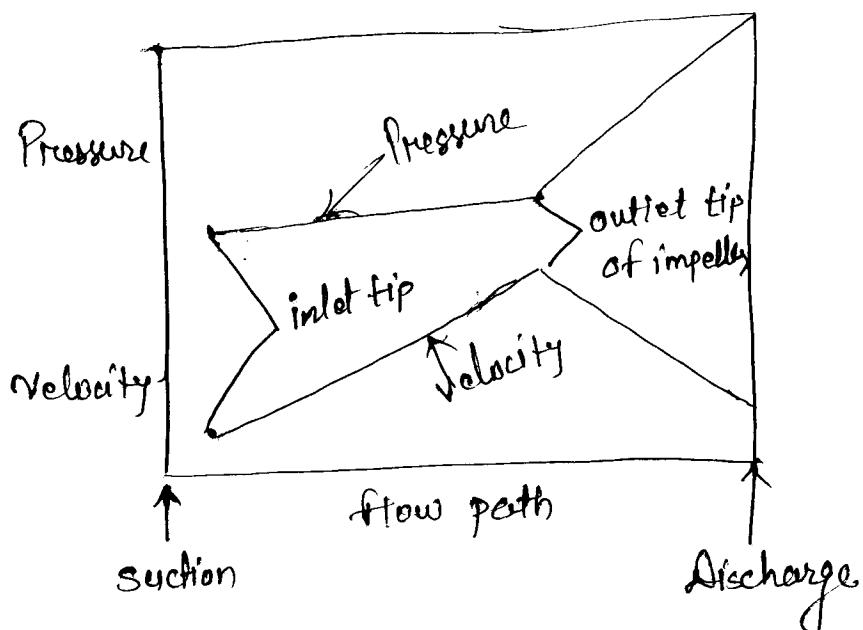
↓
Energy continuously added

↓
Energy periodically added.

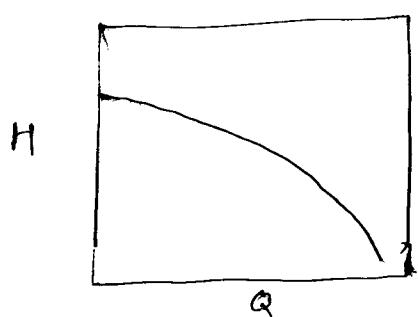
* How Centrifugal pumps work :-



Radial load :-



* H-Q Curve :-

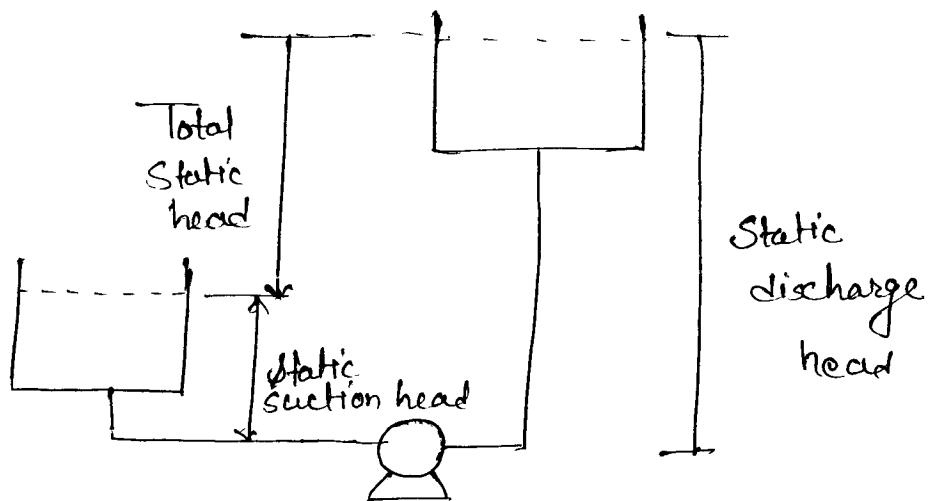


* When to choose a P.D. pumps:

- (i) High viscosity (ii) Self priming
- (iii) High pressure (iv) low flow
- (v) High efficiency (vi) low velocity

* Head:-

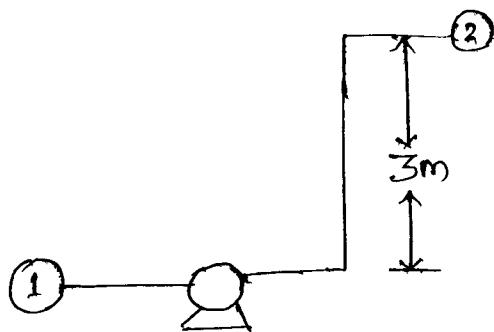
- (i) Static Head



③ Friction head:-

- (c) Pressure head
- (d) Velocity head

*



$$P_1 = 1.25 \times 10^5 \text{ Pa}$$

$$A_1 = 15 \times 10^{-4} \text{ m}^2$$

$$U_1 = 1 \text{ m/s}$$

$$P_2 = 1.05 \times 10^5 \text{ Pa}$$

$$A_2 = 5 \times 10^{-4} \text{ m}^2$$

$$U_2 = 3 \text{ m/s}$$

Power delivered by pump = 7.5W

$$\eta = 100\%$$

$$P = mg h$$

$$A_1 V_1 = A_2 V_2$$

$$V_2 = 3 \text{ m/s}$$

$$Q = 1.5 \times 10^{-3} \text{ m}^3/\text{s}$$

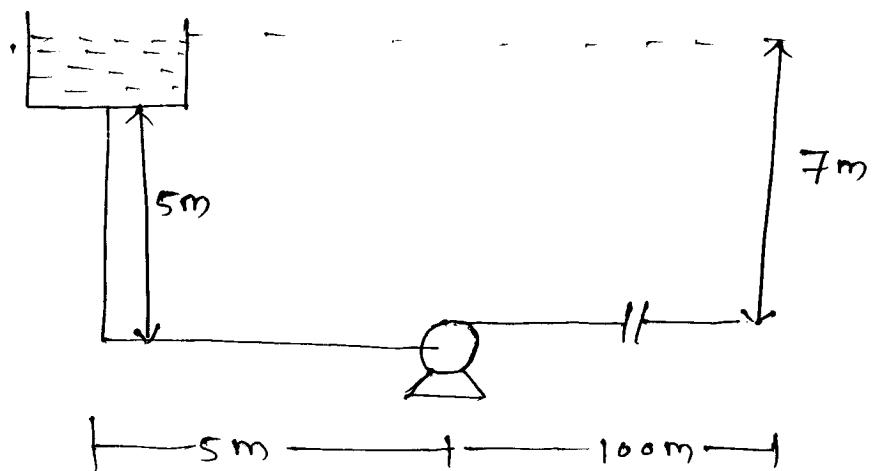
$$m = 1.5 \text{ kg/s}$$

$$\boxed{h_p = 0.51 \text{ m}}$$

$$\therefore \frac{P_1}{\rho g} + z_2 + \frac{V_1^2}{2g} + h_p = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f$$

Q.2.

$$\mu = 100 \times 10^{-3} \text{ Ns/m}^2$$



$$d_i = 7 \times 10^{-2} \text{ m}$$

$$\eta = 0.8$$

$$Q = 20 \text{ m}^3/\text{hr} = 5.55 \times 10^{-3} \text{ m}^3/\text{s}$$

$$\rho = 800 \text{ kg/m}^3$$

$$\frac{P_1}{\rho g} + z_1 + \frac{V_1^2}{2g} + h_w \eta = \frac{P_2}{\rho g} + z_2 + \frac{V_2^2}{2g} + h_f$$

$$7 + h_w \eta = h_f$$

$$h_f = \frac{4 f d u^2}{2 g D}$$

$$f = \frac{16}{R_e} = 0.019$$

$$R_e = \frac{\Delta u S}{\mu} = \frac{7 \times 10^{-2} \times 1.44 \times 800}{1000 \times 10^{-3}} = 808.4$$

$$h_f = \frac{4 \times 0.019 \times 110 \times (1.44)^2}{2 \times 9.81 \times 7 \times 10^{-2}} = 12.62$$

$$h_w \eta = 12.62 - 7 = 5.62$$

$$h_w = 7.02$$

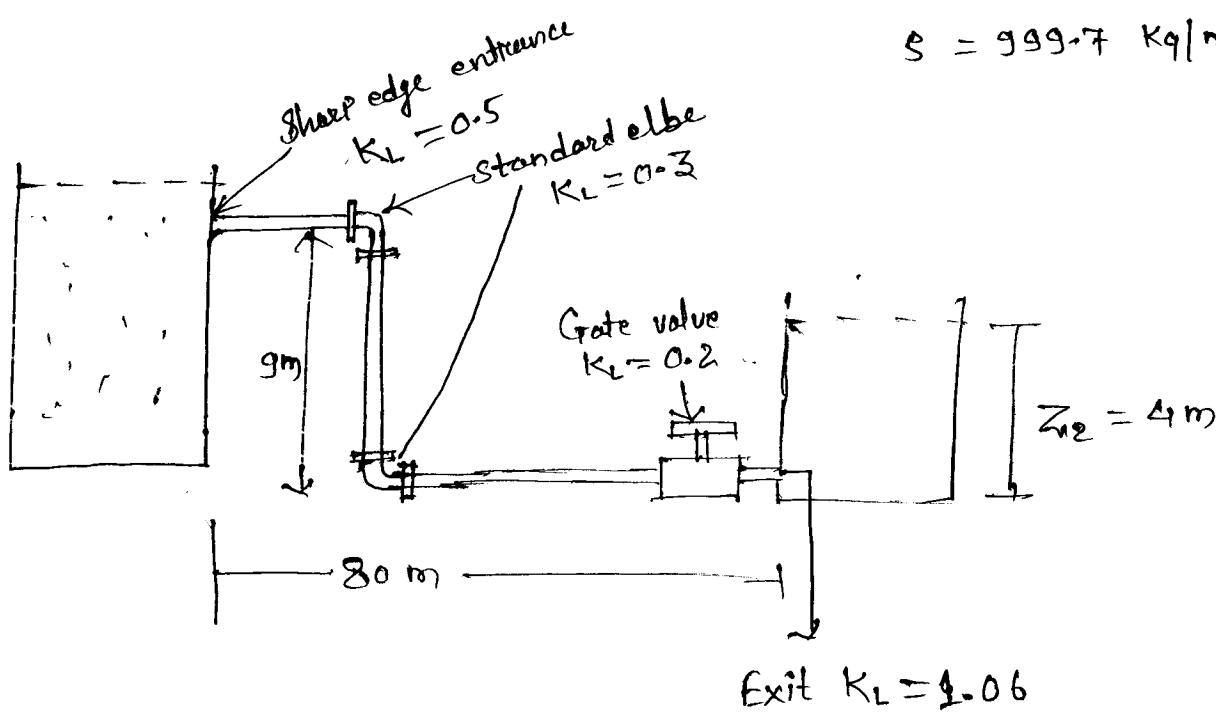
$$P = 303.34 \text{ W}$$

* Gravity-driven water flow in a pipe

- Q. Water at 10°C flows from a large reservoir to a smaller one through 5 cm diameter cast iron piping system. Determine the elevation Z_1 for a flow rate of 6 L/s.

$$\mu = 1.307 \times 10^{-3} \frac{\text{kg}}{\text{ms}}$$

$$\rho = 999.7 \text{ kg/m}^3$$



$$Q = 6 \text{ L/s} = 6 \times 10^{-3} \text{ m}^3/\text{s}$$

$$A = \pi/4 (5 \times 10^{-2})^2 = 1.96 \times 10^{-3} \text{ m}^2$$

$$u = 3.05 \text{ m/s}$$

$$Re = \frac{\rho u s}{\mu} = 117,000$$

$$f = 0.079 (Re)^{-0.25} = 4.27 \times 10^{-3}$$

$$Z_1 = Z_2 + h_f$$

$$= 4 + \left(\frac{4 f L u^2}{2 g \rho} + (K_1 + K_2 + K_3 + K_4 + K_5) \frac{u^2}{2g} \right)$$

$$Z_1 = 4 + 14.41 + 1.1189 = 19.52 \text{ m}$$