Control Systems

What
is
Transfer Function ?

Prof Pradeep SINGH HADA

Transfer Function

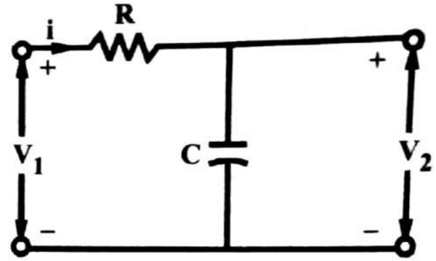
The transfer function of a linear time invariant system is defined as

“The ratio of the Laplace transform of the output variable to the Laplace transform of the input variable” under the assumption that all initial conditions to be zero.

Let G(s) denotes the transfer function of a single-input-single-output (SISO) system with input, r(t), output, y(t) and impulse response, g(t). Then the transfer function G(s) is given by

$$G(s) = \frac{Y(s)}{R(s)}$$

R(s) are the Laplace transforms of y(t) and r(t), respectively.



Consider the RC Network as shown in Fig. The Transfer Function of this RC network is obtained by KVL equation as

$$V_1(s) = \left[R + \frac{1}{C(s)}\right] I(s) \qquad \ldots(i)$$

The output voltage is given by

$$V_2(s) = I(s)\left[\frac{1}{C(s)}\right] \qquad \ldots(ii)$$

Substituting the value of I(s) from equation (i) into equation (ii), then we get

$$V_2(s) = \frac{[1/C(s)]V_1(s)}{R + 1/C(s)}$$

The transfer function G(s) is obtained as the ratio of $V_2(s)/V_1(s)$ in the form

$$G(s) = \frac{V_2(s)}{V_1(s)} = \frac{1}{RC(s)+1} = \frac{1}{\tau s+1} = \frac{(1/\tau)}{s+1/\tau} \qquad \ldots(iii)$$

where $\tau = RC$, is the time constant of the network. Let the input-output relation of a linear time invariant system is described by the n^{th} order differential equation with constant real coefficients as follows –

$$\frac{d^n y}{dt^n} + a_{n-1}\frac{d^{n-1} y}{dt^{n-1}} + \ldots + a_1 \frac{dy}{dt} + a_0 y(t)$$

$$= b_m \frac{d^m r}{dt^m} + b_{m-1}\frac{d^{m-1} r}{dt^{m-1}} + \ldots + b_1 \frac{dr}{dt} + b_0 r(t) \qquad \ldots(iv)$$

The coefficients $a_0, a_1, \ldots, a_{n-1}$ and $b_0, b_1, \ldots, b_m$ are real constant. To determine the transfer function of the linear system shown in equation (iv) and just simply take the Laplace transform on both sides of the equation with assuming zero initial conditions. Then,

$$\left(s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0\right)Y(s) = \left(b_m s^m + b_{m-1}s^{m-1} + \ldots + b_1 s + b_0\right)R(s) \quad \ldots(v)$$

The transfer function between r(t) and y(t) is obtained as

$$G(s) = \frac{Y(s)}{R(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \ldots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0}$$

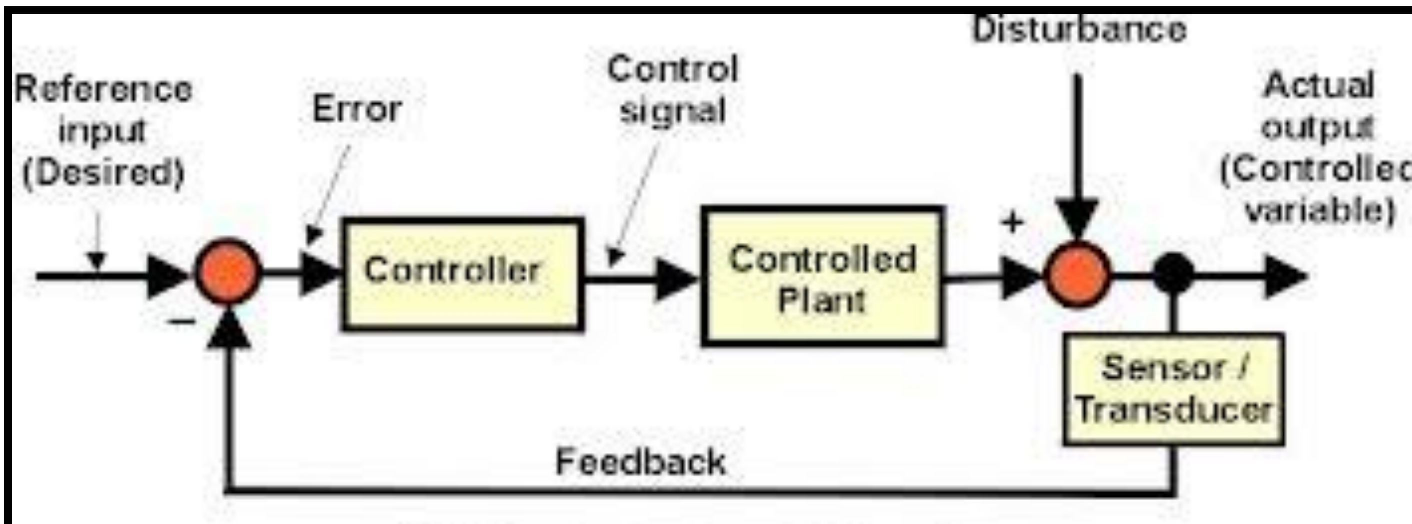
Thank You for Watching

Prof. Pradeep Singh Hada, IPS Academy, IES Indore

Closed-loop control system:

It is a control system where its control action depends on both of its input signal and output response.

Examples: automatic electric iron, missile launcher, speed control of DC motor, etc.



Practical Examples of Closed Loop Control System

- **Automatic Electric Iron** – Heating elements are controlled by output temperature of the iron.
- **Servo Voltage Stabilizer** – Voltage controller operates depending upon output voltage of the system.
- **Water Level Controller** – Input water is controlled by water level of the reservoir.
- **Missile Launched and Auto Tracked by Radar** – The direction of missile is controlled by comparing the target and position of the missile.
- **An Air Conditioner** – An air conditioner functions depending upon the temperature of the room.
- **Cooling System in Car** – It operates depending upon the temperature which it controls.

Basic block diagram of a closed-loop negative feedback control system

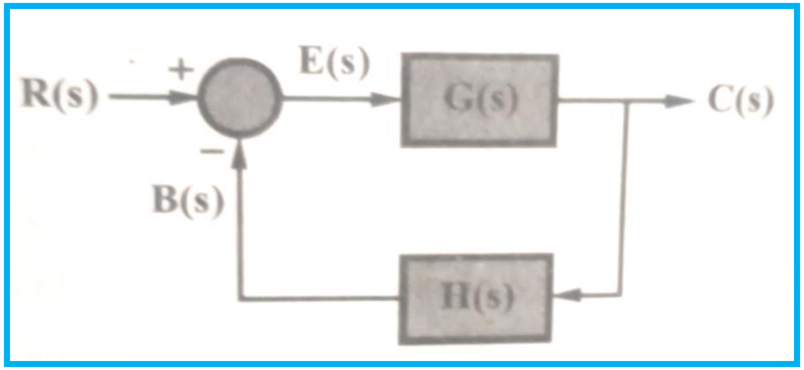
Let R(s) = Reference Input

B(s)= Feedback Signal

E(S) = Manipulated Signal

G(S) = Forward path transfer function

H(S) = Feedback path transfer function



As $$E(S) = R(s) - B(s) \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

and $$B(s)= C(s).H(S) \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

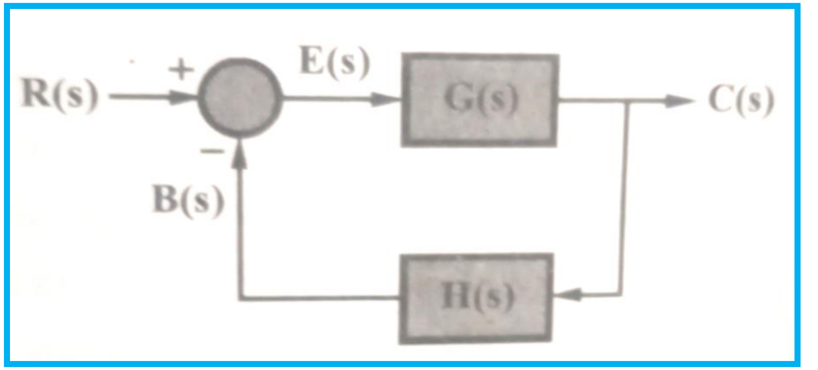
Substituting the value of B(s) from eq. (2) in equation (1) from (in), we have

$$E(S) = R(S) - C(s).H(S) \quad \ldots\ldots\ldots\ldots(3)$$

It is also seen that

$C(S) = E(s).G(s)$

Or $E(s) = \frac{C(s)}{G(s)}$ ....(4)



Put the value of E(s) from Eq. 4 in Eq.3

$$\frac{C(s)}{G(s)} = R(S) - C(s).H(s)$$

$$C(s) = G(s).\ R(S) - G(s).\ C(s).H(s)$$

$$C(s) + G(s).\ C(s).H(s) = G(s).R(S)$$

$$C(s)\ [1+G(s).H(s)] = G(s).\ R(S)$$

$$C(s) = \frac{G(s).R(S)}{[1+G(s).H(s)]}$$

$$\frac{C(s)}{R(S)} = \frac{G(s)}{[1+G(s).H(s)]}$$