Differential equation for General First order system (Derivation)

RGPV

B. Tech

4th Sem

Exam Important

Instrumentation & Control

Mechanical Engineering

Differential equation for General First order system

The general differential equation of a first order system is given by

$$a_1 \frac{dc(t)}{dt} + a_0 c(t) = b_0 r(t) \qquad \ldots(i)$$

Any system which follows this equation is by definition a first order system.

Equation (i) has three constants a_0, a_1 and b_0, but it can have only two. Thus, dividing equation (i) by a_0, we get

$$\frac{a_1}{a_0}\frac{dc(t)}{dt} + c(t) = \frac{b_0}{a_0} r(t)$$

or

$$\tau \frac{dc(t)}{dt} + c(t) = S\, r(t) \qquad \ldots.(ii)$$

$$\tau = \text{Time constant} = \frac{a_1}{a_0}$$

$$S = \text{Static sensitivity} = \frac{b_0}{a_0}.$$

The time constant t has the dimensions of time. The static sensitivity S has the dimensions of output/input. This is true not only for systems of first order but for systems of any order. This is because under static conditions all derivatives are zero and the only coefficients that remain under such conditions are a_0 and b_0.

Therefore the above definition of static sensitivity holds good for all systems.

Taking Laplace transform of equation (ii), we get

$$\tau s\, C(s) + C(s) = SR(s)$$
$$C(s)\,(1 + \tau s) = SR(s)$$

$$\tau \frac{dc(t)}{dt} + c(t) = S\, r(t)$$

Transfer function of a first order system is given by

$$G(s) = \frac{C(s)}{R(s)} = \frac{S}{1+\tau s}$$

Here transfer function has units of input/output, like N/m or V/m. To study the behavior of the system in per unit terms transfer function need to be made dimensionless which can be done by dividing it by S. Hence the dimensionless transfer function is,

$$G(s) = \frac{1}{1 + \tau s}$$

Thank You