DYNAMICS FOR SINUSOIDAL INPUT TO A FIRST ORDER SYSTEM

Instrumentation & Control

B. Tech 4th Sem

Mechanical Engineering

Assuming steady state response, solution of equation (ii) will be

$$c(t) = A \sin \omega t + B \cos \omega t \qquad \text{.........(iii)}$$

Substituting values of c(t) and r(t) in equation (i) and solving, we get

$$\tau\omega A \cos \omega t - \tau\omega B \sin \omega t + A \sin \omega t + B \cos \omega t = SR \sin \omega t$$

Equating coefficients of sin ωt and cos ωt on both sides, we get

$$\tau\omega A + B = 0 \qquad \text{.........(iv)}$$

and

$$-\tau\omega B + A = SR \qquad \text{.........(v)}$$

Solving equations (iv) and (v), we get

$$A = \frac{SR}{1+\tau^2\omega^2} \text{ and } B = \frac{-S\tau\omega R}{1+\tau^2\omega^2}$$

Putting values in equation (iii), we get

$$c(t) = \frac{SR}{1+\tau^2\omega^2} \sin \omega t + \frac{-S\tau\omega R}{1+\tau^2\omega^2} \cos \omega t \qquad \text{....(vi)}$$

Let

$$\frac{SR}{1+\tau^2\omega^2} = C \cos \phi \text{ and } \frac{-S\tau\omega R}{1+\tau^2\omega^2} = C \sin \phi$$

then equation (vi) can be written as,

$$c(t) = C \sin (\omega t + \phi)$$

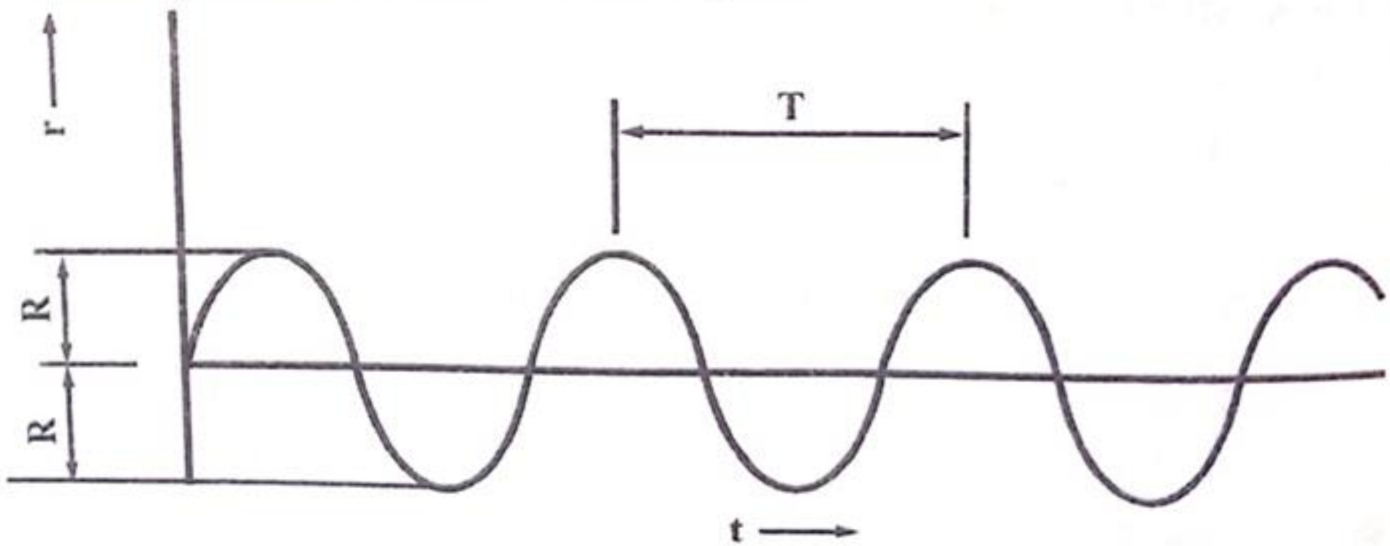
Let this system be subjected to a sinusoidal input as shown in fig. and governed by the equation,

$$r(t) = R \sin \omega t \qquad \text{..........(ii)}$$

where R = Amplitude

ω = Circular frequency = $\frac{2\pi}{T}$

T = Time period of sinusoidal input.



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