Mathematical model of First Order System for a Liquid Filled Thermometer

B. Tech 4th Sem

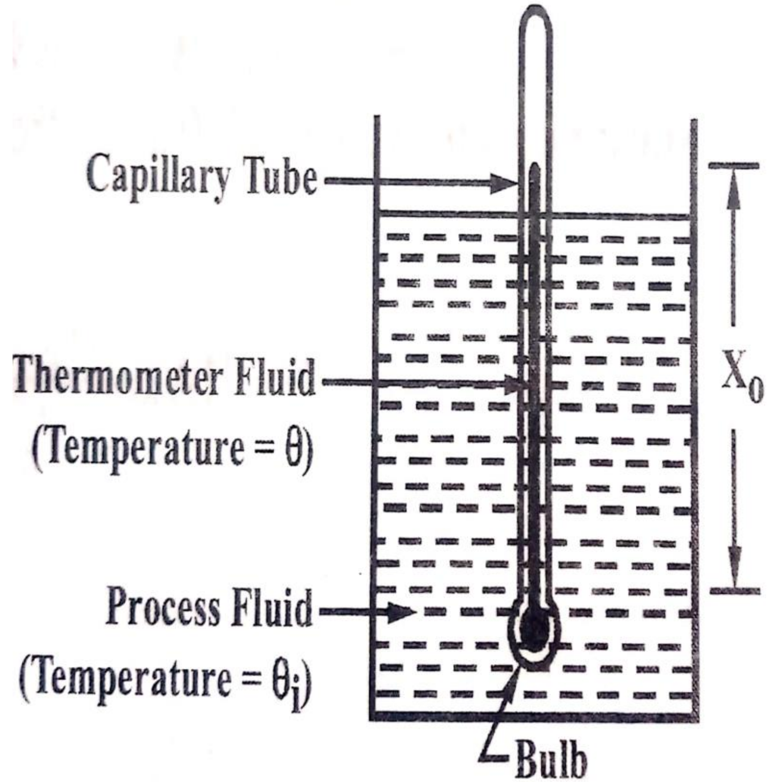
Instrumentation & Control

Mechanical Engineering

Mathematical model of First Order System for a Liquid Filled Thermometer

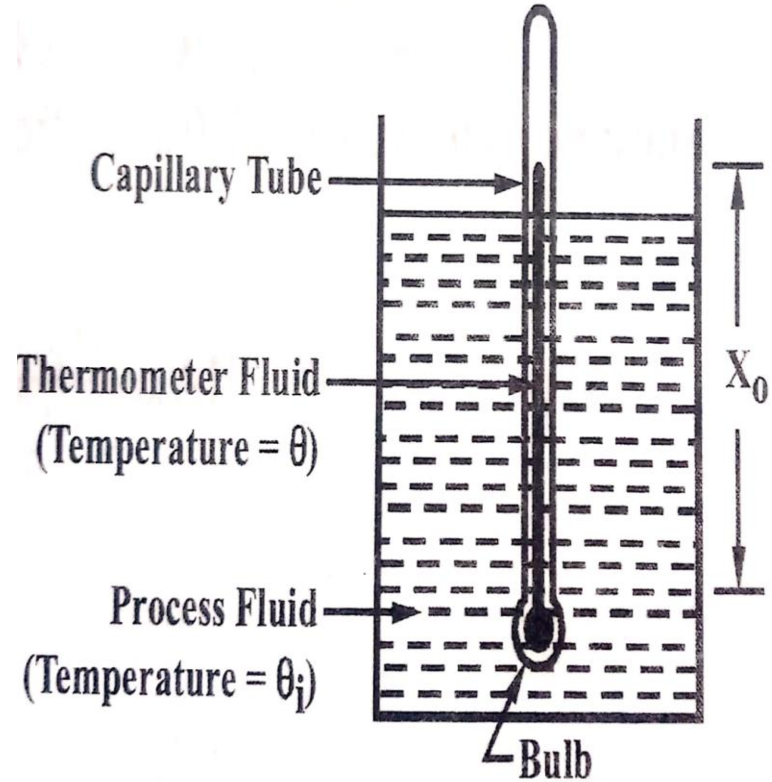
- Measurement systems that contain storage elements cannot respond instantaneously, to change in input.

- The liquid filled thermometer has a bulb, that exchanges energy with its environment until the temperature of both becomes equal and stores energy during the exchange.



The temperature of the bulb sensor will change with time until the equilibrium is reached, which accounts physically for its less than immediate response.

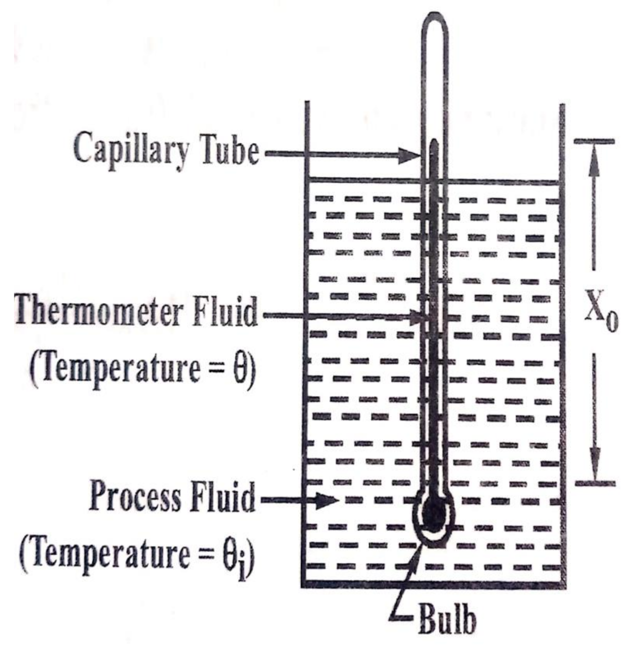
The rate at which temperature changes with time can be modeled With a first order derivative, and the thermometer behaviour can be modeled as a first order system.



A liquid filled thermometer is shown in fig. The input quantity is the temperature $\theta_i(t)$ of the fluid surrounding the bulb of the thermometer and the output is the displacement $x_o(t)$ of the thermometer fluid in the capillary tube.

The temperature is assumed to be uniform throughout the fluid at a particular time but may vary with the time.

The working of the thermometer is based upon the thermal expansion of the filling fluid which makes the liquid column rise or fall according to the temperature variations.



Let V_b = Volume of thermometer bulb

A_c = Area of capillary tube

α = Coefficient of differential expansion of thermometer bulb and fluid.

H = Coefficient of heat transfer across the bulb

A_b = Area of heat transfer in the bulb

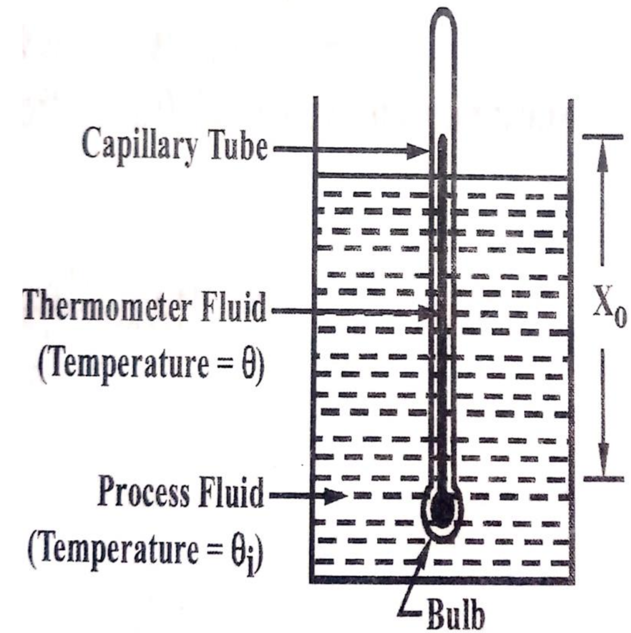
ρ = Density of thermometer fluid

s = Specific heat of thermometer fluid

θ_i = Process fluid temperature

θ = Temperature rise of fluid at any time t

x_0 = Rise of the thermometer fluid in the capillary tube.



Considering conservation of energy for the thermometer **over an** infinitesimal interval of time dt for the bulb. During this period, the **temperature** of thermometer fluid rises by $d\theta$.

Heat stored = $V_b \rho s \, d\theta$

Heat input = $HA_b(\theta_i - \theta)\, dt$

Heat output = 0, assuming no heat dissipation to outside

According to conservation of energy

$$\text{Heat stored} = \text{Heat input} - \text{Heat output}$$

$$V_b \, \rho s d\theta = HA_b(\theta_i - \theta)\, dt - 0$$

or $$V_b \rho s \frac{d\theta}{dt} + HA_b\theta = HA_b\theta_i \qquad \text{........(i)}$$

But volumetric expansion = Area × Linear expansion

$$\alpha V_b d\theta = A_c dx_0$$

or $$V_b d\theta = \frac{A_c}{\alpha} dx_0 \qquad \text{........(ii)}$$

Integrating above equation, we get

$$V_b \theta = \frac{A_c}{\alpha} x_0 \qquad \text{.........(iii)}$$

Substituting the values of $d\theta$ and θ from equations (ii) and (iii), in **eqation** (i), we get

$$V_b \rho s.\frac{A_c}{V_b.\alpha}.\frac{dx_0}{dt} + HA_b.\frac{A_c}{\alpha.V_b}x_0 = HA_b\theta_i$$

Multiplying by $\frac{V_b.\alpha}{HA_bA_c}$, we get

$$\frac{V_b.\rho s}{HA_b}.\frac{dx_0}{dt} + x_0 = \alpha.\frac{V_b}{A_c}\theta_i \qquad \textbf{............(iv)}$$

Comparing equation (iv), with the general equation of first order we get

$$\tau\frac{dc(t)}{dt} + c(t) = S\,r(t)$$

Time constant, $\tau = \frac{V_b \rho s}{HA_b}$

and static sensitivity, $S = \alpha.\frac{V_b}{A_c}$