**Gear train** - Combination of gears, to transmit power from one shaft to another..

Why gear train ??

1 Gear train is used to transmit power large centre distance b/w two shaft.

2. Require low or high velocity ratio.

A Gear train may consist of spur, bevel or spiral gears.

Types of Gear Trains.

1. Simple gear train

2. Compound gear train

3. Reverted gear train

4. Epicyclic gear train

- Each gear train follows law of gearing $\frac{\omega_1}{\omega_2} = \frac{D_2}{D_1} = \frac{T_2}{T_1}$

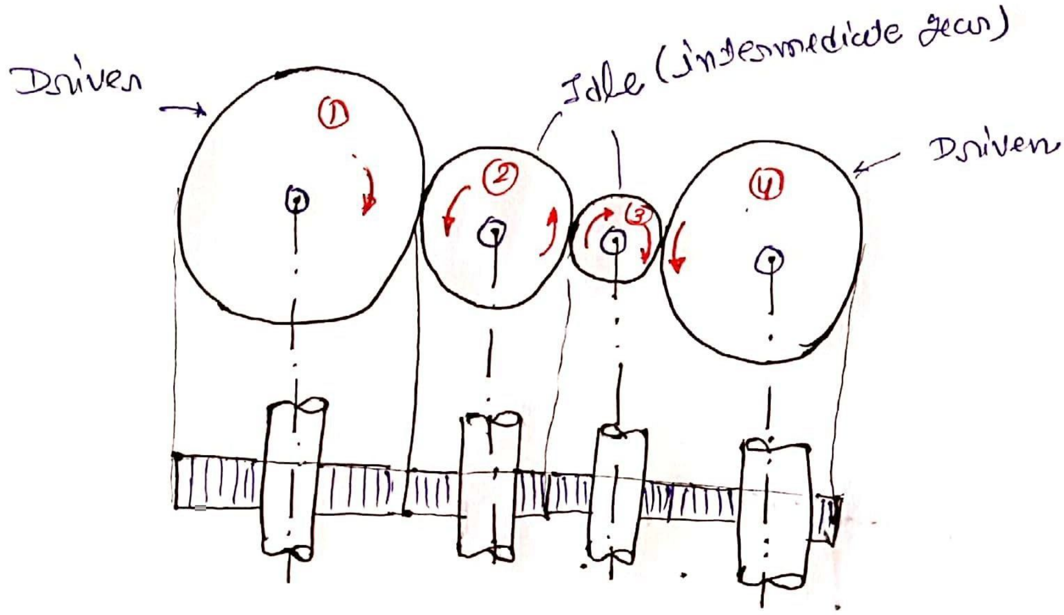
Two important term

- Speed ratio = The ratio of speed of driver to speed of driven. (follower) $SR = \frac{N_1}{N_2} = \frac{T_2}{T_1}$
- Train Value = Reciprocal of speed ratio.

$$= \frac{1}{SR} \quad \text{i.e} \quad \frac{N_2}{N_1} = \frac{T_1}{T_2}..$$

1. Simple Gear Train

- There is only one gear on each shaft
- The distance b/w the two shaft is small




Let $N_1, N_2, N_3, N_4$ = corresponding speeds on ①, ②, ③ & ④ respectively

$T_1, T_2, T_3, T_4$ = corresponding no. of teeth on gear ①, ②, ③ & ④

Now, apply law of gearing

①, ②

$$\frac{\omega_1}{\omega_2} \text{ or } \frac{N_1}{N_2} = \frac{T_2}{T_1} \quad \text{—①}$$

b/w ② & ③

$$\frac{\omega_2}{\omega_3} \text{ or } \frac{N_2}{N_3} = \frac{T_3}{T_2} \quad \text{—②}$$

b/w ③ & ④

$$\frac{\omega_3}{\omega_4} \text{ or } \frac{N_3}{N_4} = \frac{T_4}{T_3} \quad \text{—③}$$

eqn ① × ② × ③

$$\frac{\omega_1}{\omega_4} = \frac{N_1}{N_4} = \frac{T_4}{T_1} = \text{Speed ratio}$$

From the above eqn, the speed ratio is independent of size & number of intermediate gear. The intermediate gear are called Idle gears.

- If No. of gears (intermediate gears)
  - odd – dirn is same
  - even – dirn opposite

# Compound Gear Train

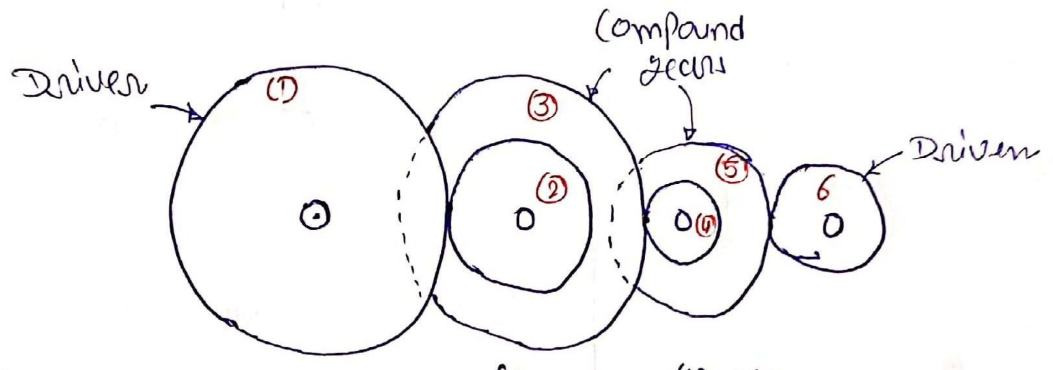
- When there are more than one gear on a shaft, it is called compound train of gear.
- In simple gear train, idle gear are used only for bridging over the space b/w driver & driven. Idle gear do not effect the speed ratio.
- Whenever, the distance b/w the driver & driven or follower bridged over by intermediate gear & at the same time a great (or much less) speed ratio is required, then advantage of intermediate gears is intensified by providing compound gears.

Modules

$m_1 = m_2$

$m_3 = m_4$

$m_5 = m_6$




$\omega_2 = \omega_3$ $\qquad$ $\omega_4 = \omega_5$.

Speed ratio b/w gear 1 & 2

$$\frac{\omega_1}{\omega_2} = \frac{T_2}{T_1} \quad \text{—(1)}$$

b/w gear (3) & (4)

$$\frac{\omega_3}{\omega_4} = \frac{T_4}{T_3}$$

b/w gear (5) & (6)

$$\frac{\omega_5}{\omega_6} = \frac{T_6}{T_5}.$$

(1) × (2) × (3)

$$\frac{\omega_1}{\omega_2} \times \frac{\omega_3}{\omega_4} \times \frac{\omega_5}{\omega_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5}.$$

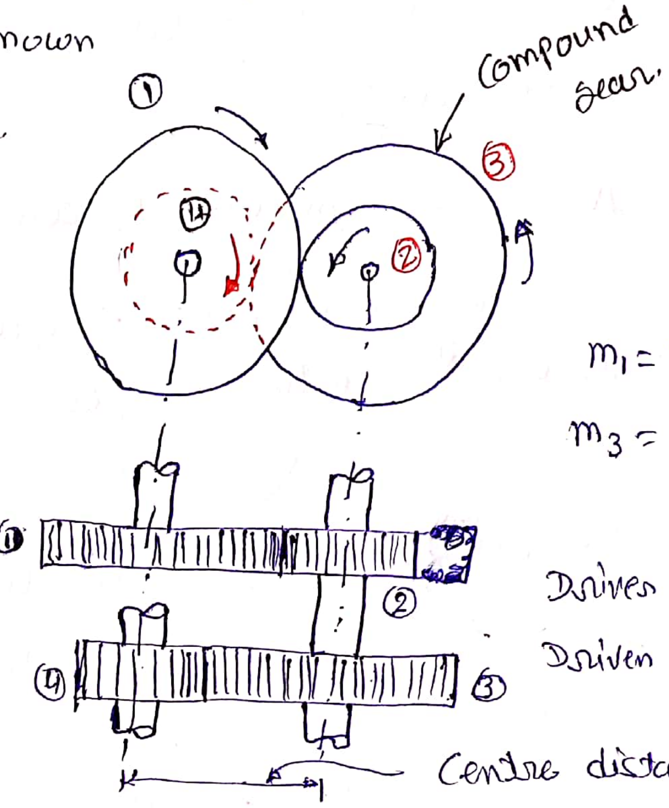
$$SR = \frac{\omega_1}{\omega_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}$$

$$= \frac{\text{Product of No. of teeth on (follower) Driven}}{\text{Product of No of teeth on Driver}}$$

Scanned with CamScanner

3. Reverted Gear Train

- When the axes of driver & follower are co-axial The gear train is known as reverted gear train.
- In reverted gear train the motion of the first & the last gear is like. (i.e same direction)




$m_1 = m_2 = m$

$m_3 = m_4 = m'$

Driver (1, 3)

Driven (2, 4)

Let $T_1, T_2, T_3$ & $T_4$ = No. of teeth on gears 1, 2, 3 & 4 respectively

$r_1, r_2, r_3$ & $r_4$ = Pitch circle radii of gears 1, 2, 3 & 4 respectively

$N_1, N_2, N_3$ & $N_4$ = Speed of gears 1, 2, 3 & 4 respective (in RPM)

Speed ratio $\dfrac{N_1}{N_4} = \dfrac{T_2 \times T_4}{T_1 \times T_3}$

NOTE :- If speed reduction is same b/w 1 & 2 & 3 & 4., then

$$\frac{T_2}{T_1} = \frac{T_4}{T_3} \quad \text{then} \quad SR = \left(\frac{T_2}{T_1}\right)^2$$

A general concept.

$$r_1 + r_2 = r_3 + r_4$$

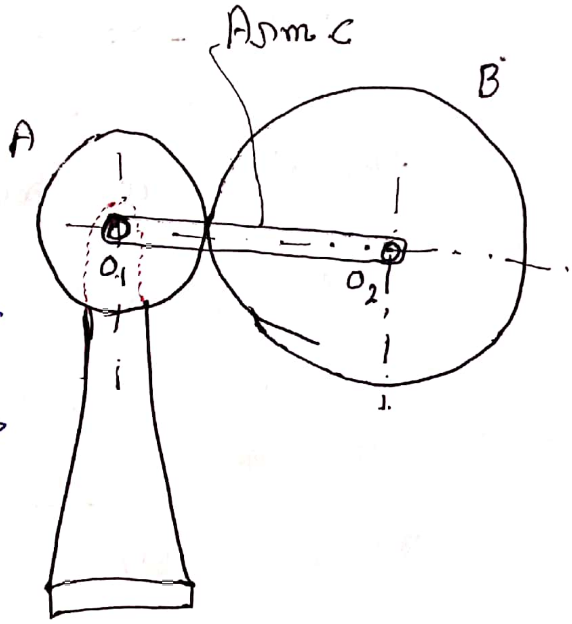
$$\frac{mT_1}{2} + \frac{mT_2}{2} = \frac{m'T_3}{2} + \frac{m'T_4}{2}$$

$$\boxed{m(T_1 + T_2) = m'(T_3 + T_4)}$$

if module is same $m = m'$

$T_1 + T_2 = T_3 + T_4$

# 4. Epicyclic Gear Train




- Gear B is forced to rotate upon and around gear A.
- Gear A & B is connected by arm C.
- Such a motion is called epicyclic that is why it is called epicyclic gear train.
- The epicyclic gear trains are useful for transmitting high velocity ratio's with gears to moderate size in comparatively lesser space.
- Epicyclic gear train are used in the back gear of lathe, differential gears of the automobiles, wrist watches etc.

The velocity ratios of epicyclic gear train, finding out by two methods. 1. Tabular method 2. Algebraic method.

1 Tabular method ACW(+Ve) & CW(-Ve)

| Step No. | Condition of motion | Revolution of Elements. | | |
|---|---|---|---|---|
| | | Arm C | Gear A | Gear B |
| 1. | Arm fixed, Gear A rotate through +1 revolution (ACW) | 0 | +1 | $-\frac{T_A}{T_B}$ |
| 2 | Arm fixed, Gear A rotate through +x revolution | 0 | +x | $-x \times \frac{T_A}{T_B}$ |
| 3. | Add +Y revolution of all elements | +Y | Y+x | $Y - x\frac{T_A}{T_B}$ |
| | | | | |

Example :- In an epicyclic gear train, an arm carries two gears A & B having 36 & 45 teeth respectively. If the arm rotates at 150 r.p.m. in anticlockwise direction about the centre of the gear A, which is fixed, determine the speed of gear B. If the gear A, instead of being fixed, makes 300 r.p.m. in the clockwise direction, what will be the speed of gear B.?

Solution :- By Tabular method.

Given :- $T_A = 36$, $T_B = 45$ ; $N_C = 150$ rpm

| Step No. | Condition of motion | Revolution of Elements. | | |
|---|---|---|---|---|
| | | Arm C | Gear A | Gear B |
| 1. | Arm fixed, gear A rotate through +1 Revolution (ACW) | 0 | +1 | $-\frac{T_A}{T_B}$ |
| 2. | Arm fixed, gear A, $+x$ revolution | 0 | $+x$ | $-x \cdot \frac{T_A}{T_B}$ |
| 3. | Add $+y$ rev. of all element | $+y$ | $x+y$ | $y - x \cdot \frac{T_A}{T_B}$ |

Case - I.

Speed of gear B, when gear A fixed

Speed of arm $N_C$ (Given) $= 150$ r.p.m.

i.e $\quad y = 150$

From last row, do all calculation.

gear A fixed, $x + y = 0 \quad x = -y$

$x = -150$ r.p.m.

Speed of gear B. $y - x \frac{T_A}{T_B} = 150 - (-150) \times \frac{36}{45}$

$= +270$ r.p.m

$= 270$ r.p.m (ACW)

Case. 2. Gear A makes 300 rpm (CW)

$x + y = -300$

$x = -300 - 150 = -450$

Speed of gear B

$= y - x \frac{T_A}{T_B}$

$= 150 + \left(450 \times \frac{36}{45}\right)$

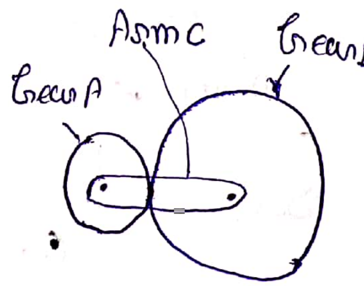
$= +510$ r.p.m.

$= 510$ (ACW).

# Algebraic method

- The motion of each element of epicyclic train relative to the arm is set down in the form of equations.
- The No. of equation depends on No. of elements in gear train.

* - Two conditions are sufficient to determine motion of all elements.
  - Some element is fixed.
  - Other element has specified motion.




Let the arm C be fixed, therefore speed of gear A relative to the arm C. $= N_A - N_C$

& speed of gear B relative to arm C $= N_B - N_C$

Since the gear A & B are meshing directly, therefore they will revolve in opposite directions.

$$\therefore \quad \frac{N_B - N_C}{N_A - N_C} = -\frac{T_A}{T_B}.$$

Since the arm C is fixed, therefore $N_C = 0$

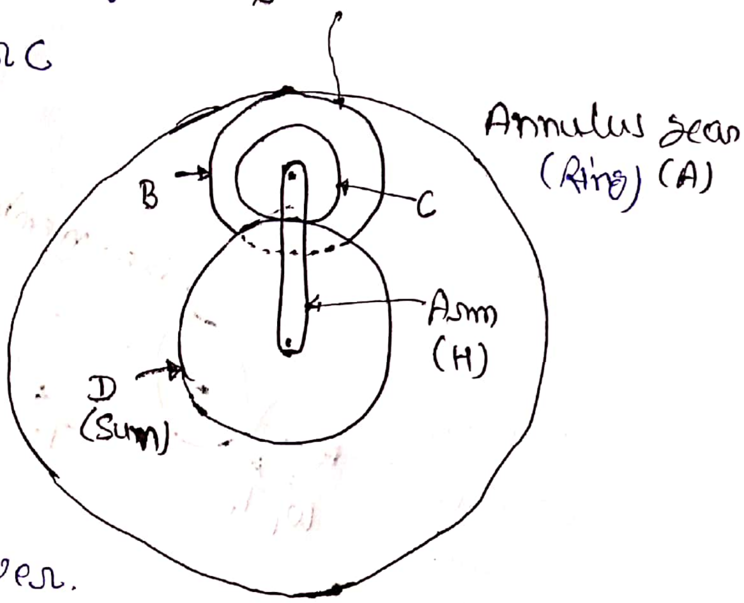
$$\therefore \quad \boxed{\frac{N_B}{N_A} = -\frac{T_A}{T_B}.}$$

If gear A is fixed, then $N_A = 0$

$$\frac{N_B - N_C}{0 - N_C} = -\frac{T_A}{T_B} \quad \text{or} \quad \boxed{\frac{N_B}{N_C} = 1 + \frac{T_A}{T_B}}$$

# 5. Compound Epicyclic Gear Train – Sun and Planet Gear.

- The annulus gear A meshes with gear B and sun gear (D) meshes with gear C
- When annulus gear is fixed, the sun provides the drive
- When the sun gear is fixed, the annulus gear provides the drive. ~~Tee~~
- The arm acts as a follower.



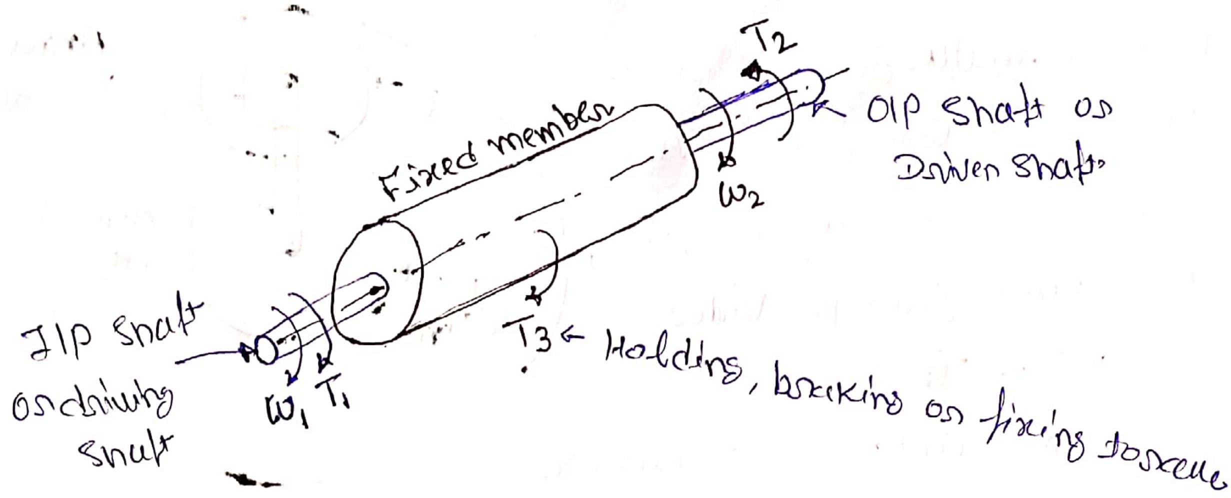

- Let $T_A, T_B, T_C$ & $T_D$ → No. of teeth on gear A, B, C & D respectively

| Step No | Condition of motion | Revolution of Elements | | | |
|---|---|---|---|---|---|
| | | Arm(H) | Gear D | Compound Gear (B/C) | Gear A |
| 1. | Arm fixed - gear D rotates through +1 rev (ACW) | 0 | +1 | $-\frac{T_B}{T_C}$ | $-\frac{T_D}{T_C} \times \frac{T_B}{T_A}$ |
| 2. | Arm fixed - gear D rotates through $+x$ revolution | 0 | $+x$ | $-x\frac{T_D}{T_C}$ | $-x \cdot \frac{T_D}{T_C} \times \frac{T_B}{T_A}$ |
| 3. | Add $+y$ rev. to all elements | $+y$ | $x+y$ | $y-x\frac{T_D}{T_C}$ | $y-x\cdot\frac{T_D}{T_C}\cdot\frac{T_B}{T_A}$ |

NOTE:- If the annulus gear A rotates through one rev. (ACW) with arm fixed, then compound gear rotates through $T_A/T_B$ revolution in the same sense & sun gear D rotates through $\frac{T_A}{T_B} \times \frac{T_C}{T_D}$ revolution in CW direction.

# Torques in Epicyclic Gear Trains.

Fixing or Holding torque




→ No angular acceleration, the gear train is kept in equilibrium by the three externally applied torques.

$$\Sigma T = T_1 + T_2 + T_3 = 0 \quad \text{——} (1)$$

$$\therefore F_1 r_1 + F_2 r_2 + F_3 r_3 = 0 \quad \text{——} (2)$$

$F_1, F_2, F_3$ ← applied force
$r_1, r_2, r_3$ ← radii

If $\omega_1, \omega_2$ & $\omega_3$ are angular speeds, then net K.E. dissipated by the gear train must be zero. i.e

$$T_1 \cdot \omega_1 + T_2 \omega_2 + T_3 \omega_3 = 0 \quad \text{——} (3)$$

But, for a fixed member $\omega_2 = 0$

$$\boxed{T_1 \omega_1 + T_2 \omega_2 = 0} \quad \text{——} (4)$$

From eqn (1) & (4), the holding torque or braking torque may be obtained

$$T_2 = -T_1 \times \frac{\omega_1}{\omega_2} \quad \& \quad T_3 = -(T_1 + T_2)$$

$$\boxed{T_3 = T_1\left(\frac{\omega_1}{\omega_2} - 1\right) = T_1\left(\frac{N_1}{N_2} - 1\right)}$$